



Department of Mathematical Sciences

Examination paper for **TMA4120 Matematikk 4K**

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Permitted examination support material: C: Bestemt, enkel kalkulator tillatt.

Other information:

- The problems 1, 2, 3, 4, 5, 6 have equal weight.

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Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig ☐ 2-sidig ☒sort/hvit ☒ farger ☐skal ha flervalgskjema ☐

Date

Signature

Problem 1

- a) Use Laplace transform to compute the convolution product $\cosh t \star \sin t$ for $t \geq 0$.
- b) Solve the initial value problem

$$\begin{cases} y' + y \star \cos t = 0, \\ y(0) = 1. \end{cases}$$

Problem 2 Find the Fourier series of the 2π -periodic even expansion of the function $f(x) = e^x$ for $t \in [0, \pi]$.

Problem 3

1. Find all the solutions of the partial differential equation

$$u_{xx}(x, t) = 2u_{tt}(x, t) + 2u_t(x, t) \quad \text{for } x \in [0, \pi] \text{ and } t \geq 0, \quad (1)$$

of the form $u(x, t) = F(x)G(t)$ that satisfies the border conditions

$$u(0, t) = 0 = u(\pi, t) \quad \text{for } t \geq 0.$$

2. Find a solution from (1) that satisfies the initial conditions

$$u(x, 0) = \sin(2x) \quad \text{and} \quad u_t(x, 0) = 0.$$

Problem 4

1. State the Cauchy-Riemann equations.
2. Determine a function $u(x, y)$ such that the function

$$f(x + iy) = u(x, y) + i(x^2 - y^2 + x + 1)$$

is analytic in $\mathbb{C}$.

Problem 5

1. Find and classify all the singularities of the function

$$f(z) = e^{1/z} + \frac{z}{z^2 - 1}.$$

2. Find the two Laurent series of $f(z)$ with center $z_0 = 0$

Problem 6

1. Compute the integral

$$\oint_{\mathcal{C}} \frac{z+1}{z^3 - 2z^2} dz$$

where $\mathcal{C} = \{e^{i\theta} : t \in [0, 2\pi]\}$.

2. Compute the principal value of the integral

$$\int_{-\infty}^{\infty} \frac{x+1}{x^3 - 2x^2} dx$$