



Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4140 Diskret Matematikk**

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Problem 1 Let the sequence of numbers $\{a_i\}, i \geq 0$ be given such that $a_0 = 1$, $a_1 = 3$, and $a_2 = 14$, and $a_n = 6a_{n-3} - 11a_{n-2} + 6a_{n-1}$ for $n \geq 3$.

a) Find a_3 , a_4 , and a_5 .

b) Prove by induction that $a_n = 5/2 - 5(2^n) + 7/2(3^n)$ for all n .

Problem 2 Let $\mathbb{Z}_{60} = \{0, 1, 2, 3, \dots, 59\}$ and let $f : \mathbb{Z}_{60} \rightarrow \mathbb{Z}_{60}$ be the function given by $f(a) = (12a + 3) \bmod 60$ for $a \in \mathbb{Z}_{60}$.

a) Prove that f is not a bijection and find the image of f .

b) Let $\sim_f$ be the relation on $\mathbb{Z}_{60}$ given by $a \sim_f b$ if and only if $f(a) = f(b)$. Prove that $\sim_f$ is an equivalence relation on $\mathbb{Z}_{60}$ and find the equivalence classes of $\sim_f$.

Problem 3 Let $M = (S, I, s_0, f, F \subseteq S)$ be the deterministic finite state automaton, where $S = \{s_0, s_1, \dots, s_7\}$, $I = \{0, 1, 2\}$, f is given as in figure 1, and where the final states F of M are not given.

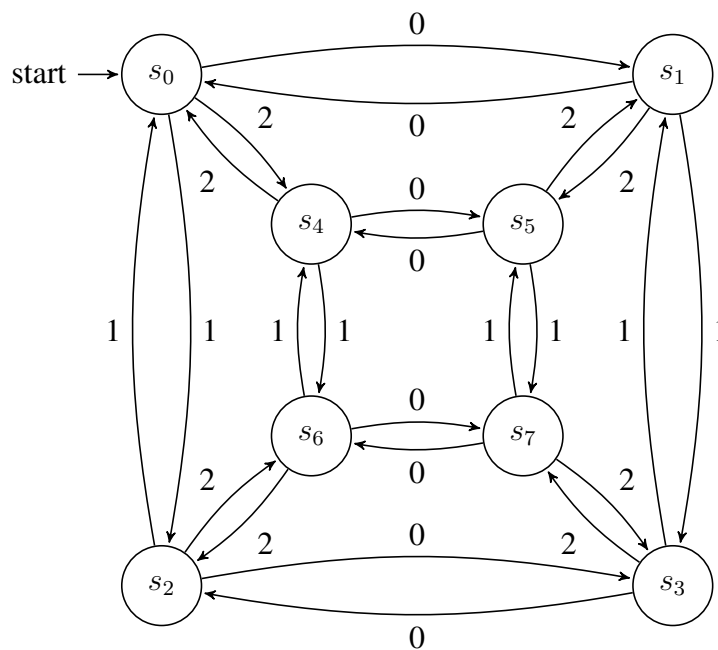


Figure 1: The finite state automaton M .

- a) Determin what F has to be in order for the language $L(M)$, which M recognizes, to consist of all strings with an even number of 0's, an odd number of 1's and an even number of 2's.
- b) How many strings (words) of length 6, and of length 7 exist in the language $L(M)$ given in a).

Problem 4 Let the simple nondirected graph G with vertices $\{a_1, a_2, a_3, \dots, a_{10}\}$ be given by the following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

and let the graph G' with vertices $\{b_1, b_2, b_3, \dots, b_{10}\}$ be given by the following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

- a) Determind if G og G' are isomorphic. You have to give a reason for your claim.
- b) Prove that the graph G' given above has a Hamilton circuit and that the graph G does not have a Hamilton circuit.

Problem 5

- a) In the expressions written in prefix order, all numbers are single digit numbers in the number 10 system, $+$ represents addition, $*$ represents multiplication, $\uparrow$ represents the exponential function, $/$ represents division, and $-$ represents subtraction. Consider the expressions $+ - * 235 / \uparrow 234$ and $* - + 264 \uparrow / 423$ in prefix notation and calculate these expressions.

- b) Find $x \in \mathbb{Z}_{135} = \{0, 1, 2, \dots, 134\}$ which satisfies the following two equivalence relations

$$x \equiv + - * 235 / \uparrow 234 \pmod{5}$$

$$x \equiv * - + 264 \uparrow / 423 \pmod{27}.$$