# Fasit MAT 121 H2017

**1.1** Augmentert matrise for systemet:

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & a & b \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 1 \\ 0 & 3 & 2 & -1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & -1 & 2 & 2a-1 & 2b-1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 3 & 2 & -1 & 1 \\ 0 & -1 & 2 & 2a-1 & 2b-1 \end{array}\right] \sim$$

$R_2 \to 2R_2 - R_1$
$R_4 \to 2R_4 - R_1$

$R_2 \leftrightarrow R_3$

$$\sim \left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & -4 & -4 & -2 \\ 0 & 0 & 4 & 2a & 2b \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & -4 & -4 & -2 \\ 0 & 0 & 0 & 2a-4 & 2b-2 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$
$R_4 \to R_4 + R_2$

i) Når $2a-4=0$ og $2b-2 \neq 0$ $(\underline{a=2}) \wedge (\underline{b \neq 1})$ systemet har <u>ingen løsning</u> siden den har en ligning av type $0=k$, $k \neq 0$.

ii) For $\underline{a=2,\ b=1}$ siste rekke er ekvivalent til $0=0$ og systemet har en fri variabel og den er kompatibel: <u>$\infty$-mange løsning</u>.

iii) For $\underline{a \neq 2}$, alle kolonner er pivot kolonner og ingen rekker av type $[0 \dots 0 | k]$ med $k \neq 0$: <u>entydig løsning</u>.

**1.2** $a=2,\ b=1$.

$$\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1, x_2, x_3$ ledende variabler
$x_4$ fri.

$$x_3 + x_4 = \frac{1}{2} \Rightarrow x_3 = \frac{1}{2} - x_4$$

$$x_2 + 2x_3 + x_4 = 1 \Rightarrow x_2 = 1 - 2x_3 - x_4 = 1 - 1 + 2x_4 - x_4 = x_4$$

$$2x_1 + x_2 + x_4 = 1 \Rightarrow x_1 = \frac{1}{2} - \frac{1}{2}x_2 - \frac{1}{2}x_4 = \frac{1}{2} - \frac{1}{2}x_4 - \frac{1}{2}x_4 = \frac{1}{2} - x_4.$$

generelle løsning: $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

(1.3) $\mathrm{Nul}\, A = \{ \underline{x} \mid A\underline{x} = \underline{0} \}$

For $a=2$: $\mathrm{Nul}\, A = \mathrm{Span}\left\{ \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$ $(\dim \mathrm{Nul}\, A = 1)$

(1.4) $\mathrm{Rank}\, A = \dim \mathrm{Col}\, A = 3$ for $\underline{\underline{a=2}}$ (3 pivot Kolonner).

En basis for Col A er pivotKolonnene av den uredusert matrise:

$$\underline{\underline{\mathrm{Col}\, A = \mathrm{Span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} \right\}}}$$

(2.1) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x - y + 2z = 0 \right\} = \mathrm{Nul}\, B$

der $B = [1 \ -1 \ 2]$ ($1 \times 3$ matrise)

Her Kan man sjekke direkte eller benytte argumentet at Nul B er et underrom av $\mathbb{R}^3$.

Sjekker direkte:

$W \subseteq \mathbb{R}^3$. Det er nok å vise at

$$\left.\begin{array}{l} \underline{v}_1, \underline{v}_2 \in W \\ c_1, c_2 \in \mathbb{R} \end{array}\right. \Rightarrow c_1 \underline{v}_1 + c_2 \underline{v}_2 \in W.$$

Dermed, anta at $\underline{v}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \in W \quad i = 1, 2.$

$$c_1 \underline{v}_1 + c_2 \underline{v}_2 = \begin{bmatrix} c_1 x_1 + c_2 x_2 \\ c_1 y_1 + c_2 y_2 \\ c_1 z_1 + c_2 z_2 \end{bmatrix}$$

$$(c_1 x_1 + c_2 x_2) - (c_1 y_1 + c_2 y_2) + 2(c_1 z_1 + c_2 z_2) =$$
$$= c_1 (x_1 - y_1 + 2z_2) + c_2 (x_2 - y_2 + 2z_2) = 0$$

Siden $x_1 - y_1 + 2z_1 = 0$ $(\underline{v}_1 \in W)$ og $x_2 - y_2 + 2z_2 = 0$ $(\underline{v}_2 \in W)$.

2.2 Basis for $W = \text{Nul } B$:

$[1 \ -1 \ 2]$ er allerede rekkeredusert.

$x$ ledende
$y, z$ fri

$x = y - 2z$

generelle løsning: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y - 2z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

Basis for $W = \underline{\underline{\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}}} = \{ \underline{b}_1, \underline{b}_2 \}$

2.3 $\underline{b}_1 \cdot \underline{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} = -2 \neq 0$ dermed $\underline{b}_1 \not\perp \underline{b}_2$.

Bruker GS:

$$\underline{\tilde{b}}_2 = \underline{b}_2 - \text{proj}_{\underline{b}_1} \underline{b}_2 = \underline{b}_2 - \frac{\underline{b}_1^T \underline{b}_2}{\underline{b}_1^T \underline{b}_1} \underline{b}_1$$

$$= \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \frac{(+2)}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}}$$

Vi ser at $\underline{\tilde{b}}_2 \in W$ (sjekk) og at $\underline{\tilde{b}}_2 \perp \underline{\tilde{b}}_1 = \underline{b}_1$

dermed en ortogonal basis for $W$ er

$$\{ \underline{\tilde{b}}_1, \underline{\tilde{b}}_2 \} = \underline{\underline{\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}}}$$

(2.4)

$$\underline{v} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad ; \quad \underline{y} = \text{proj}_W \underline{v} = \frac{\tilde{\underline{b}}_1^T \underline{v}}{\tilde{\underline{b}}_1^T \tilde{\underline{b}}_1} \tilde{\underline{b}}_1 + \frac{\tilde{\underline{b}}_2^T \underline{v}}{\tilde{\underline{b}}_2^T \tilde{\underline{b}}_2} \tilde{\underline{b}}_2$$

$$\tilde{\underline{b}}_1^T \underline{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = 3 - 1 = 2$$

$$\tilde{\underline{b}}_2 \cdot \tilde{\underline{b}}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 3$$

$$\tilde{\underline{b}}_2 \cdot \underline{v} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = -3 - 1 + 2 = -2$$

$$\underline{y} = \frac{\cancel{2}^{1}}{\cancel{2}_{1}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5/3 \\ 1/3 \\ -2/3 \end{bmatrix}}}$$

$9 - 5 = 4$

$$\underline{r} = \underline{v} - \underline{y} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 5/3 \\ 1/3 \\ -2/3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 4/3 \\ -4/3 \\ 8/3 \end{bmatrix}}}$$

$$\left( \underline{r} \cdot \tilde{\underline{b}}_1 = 0 \qquad \underline{r} \cdot \tilde{\underline{b}}_2 = 0 \quad \checkmark \right)$$

(3.1)

$$\begin{vmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 2(4-1) = \underline{\underline{6}}$$

(utregning m.h.t. 2. kolonne)

Siden $\det C = 6 \neq 0$, $\{\underline{c}_1, \underline{c}_2, \underline{c}_3\}$ er en basis for Col $C$ (invertible matrix theorem).

(3.2)

$$[\underline{v}]_{\mathcal{C}} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad : \quad \underline{v} = v_1 \underline{c}_1 + v_2 \underline{c}_2 + v_3 \underline{c}_3$$

dvs at $[\underline{v}]_{\mathcal{C}}$ er løsningen av ligningssystemet

$$C \underline{x} = \underline{v} .$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 1 & 2 & 1 & -1 \\ 1 & 0 & 2 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 4 & 0 & -5 \\ 0 & 0 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 0 & 4 & 1 & -5 \\ 0 & 0 & 3 & 1 \end{array}\right]$$

$x_3 = \frac{1}{3}$ ; $4x_2 + x_3 = -5$

$$4x_2 = 5 - \frac{1}{3} = \frac{-16}{3} \Rightarrow x_2 = -\frac{4}{3}$$

$$2x_1 = 3 - x_3 = 3 - \frac{1}{3} = \frac{8}{3} \Rightarrow x_1 = \frac{4}{3}$$

$$[\underline{v}]_{\mathcal{C}} = \underline{\underline{\begin{bmatrix} 4/3 \\ -4/3 \\ 1/3 \end{bmatrix}}}.$$

3.3 De oppgitte koordinatene er $[\underline{v}]_{\mathcal{C}}$ dermed det ønskede vektor er $\underline{v}$.

3.4 C er ikke ortogonalt diagonaliserbar siden C er ikke symmetrisk ( A ort. diagon. $\Leftrightarrow A = A^T$).

3.5
$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)\left((2-\lambda)^2 - 1\right) = 0$$

$\Rightarrow (2-\lambda) = 0 \qquad \Rightarrow \lambda = 2$

$\left((2-\lambda)^2 - 1\right) = 0 \qquad 2-\lambda = \pm 1, \quad \lambda = 2 \pm 1$

Vi har $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$

(alle egenverdiene er distinkte dermed C er diagonaliserb.)

$\lambda = 1$: $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad$ $x_3$ fri $\Rightarrow x_2 = 0$, $x_1 = -x_3$ $\quad$ $\underline{v}_1 = \underline{\underline{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}}$

$\underline{\lambda = 2}$: $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} x_3 = 0 \\ x_1 = 0 \\ x_2 \text{ fri} \end{array}$

$$\underline{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\underline{\lambda = 3}$: $\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{array}{l} x_3 \text{ fri} \\ x_1 = x_3 \\ x_2 = 2x_3 \end{array}$ $\underline{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

diagonalisering: $C = VDV^{-1}$

$$V = [\underline{v}_1, \underline{v}_2, \underline{v}_3] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3.6 $\mathcal{D} = \{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ $[\underline{w}]_{\mathcal{D}} = \underline{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Fra definisjonen av Koord. mht en basis:

$$\underline{w} = 1 \cdot \underline{v}_1 + 0 \cdot \underline{v}_2 + 0 \cdot \underline{v}_3 = \underline{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$T(\underline{w}) = C\underline{w} = C\underline{v}_1 = \lambda_1 \underline{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

siden $\lambda_1 = 1$

og $\underline{w} = \underline{v}_1$ $(C\underline{v}_1 = \lambda_1 \underline{v}_1)$

4.1 $y(t) = a + bt$

$$a + 0 \cdot b = 9.0$$
$$a + 1 \cdot b = 9.8$$
$$a + 2 \cdot b = 11.3$$
$$a + 3 \cdot b = 8.8$$

4 lign. i 2 ukjente $(a, b)$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 9.0 \\ 9.8 \\ 11.3 \\ 8.8 \end{bmatrix}}_{\underline{b}}$$

minste kvadr. problem

Normale likningene: $A^T A \underline{x} = A^T \underline{b}$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix}$$

$$A^T \underline{b} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 9.0 \\ 9.8 \\ 11.3 \\ 8.8 \end{bmatrix} = \begin{bmatrix} 9.0 + 9.8 + 11.3 + 8.8 \\ 0 + 9.8 + 22.6 + 26.4 \end{bmatrix} = \begin{bmatrix} 38.9 \\ 58.8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 38.9 \\ 58.8 \end{bmatrix}$$

$$\begin{array}{rrr} 12 & 28 & 117.6 - \\ -12 & -18 & 116.7 \\ \hline 0 & 10 & 0.9 \end{array}$$

$$\left[\begin{array}{cc|c} 4 & 6 & 38.9 \\ 6 & 14 & 58.8 \end{array}\right] \sim \left[\begin{array}{cc|c} 4 & 6 & 38.9 \\ 0 & 10 & 0.9 \end{array}\right]$$

$$b = 0.09$$
$$4a + 6b = 38.9$$
$$4a = \frac{3890}{100} - \frac{6 \cdot 9}{100} = \frac{3890 - 54}{100} = \frac{3836}{100} = 38.36$$

$$a = \frac{38.36}{4} = 9.59$$

Dermed $y(t) = 9.59 + 0.0\ t \Rightarrow y(4) = 9.59 + 0.09 \cdot 4$

$= 9.59 + 0.36$

$= \underline{\underline{9.95}}$

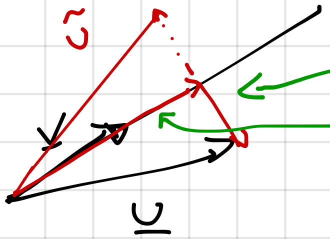
Forventet gjennomsnitt sommertemperatur for sommer 2016 er 9.95 °C ifølge modellen

Målt gjennomsnitt sommertemperatur (YR.no) for 2016 var $\underline{9.9}$ °C. Merk hvor nøyakig modellen er, med en feil av 0,05°C

Hadde man tatt kun gjennomsnitt av forrige måling, da ville estimaten vært 9.725 °C, med en feil av 0.175 °C, mer en 3 ganger større.

Siden $b = 0.09 > 0$, modellen også forteller oss at det er forventet en mild økning av temperatur m/ tid.

5.1




Dekomponér $\underline{u} = \text{proj}_{\underline{v}}\underline{u} + \underline{r}$ der $\underline{r} \perp \underline{v}$

$$\text{proj}_{\underline{v}}\underline{u} = \frac{\underline{u}^T\underline{v}}{\underline{v}^T\underline{v}}\underline{v} \qquad \underline{r} = \underline{u} - \text{proj}_{\underline{v}}\underline{u} = \underline{u} - \frac{\underline{u}^T\underline{v}}{\underline{v}^T\underline{v}}\underline{v}$$

Da: $$\tilde{\underline{u}} = \text{proj}_{\underline{v}}\underline{u} - \underline{r} = \frac{\underline{u}^T\underline{v}}{\underline{v}^T\underline{v}}\underline{v} - \left(\underline{u} - \frac{\underline{u}^T\underline{v}}{\underline{v}^T\underline{v}}\underline{v}\right) = \underline{\underline{2\frac{\underline{u}^T\underline{v}}{\underline{v}^T\underline{v}}\underline{v} - \underline{u}}}$$

5.2

skalarproduktet er kommutativ, dvs $\underline{u}^T\underline{v} = \underline{v}^T\underline{u}$

Fra 5.1 $$\tilde{\underline{u}} = 2\frac{\underline{u}^T\underline{v}}{\underline{v}^T\underline{v}}\underline{v} - \underline{u} = 2\frac{\underline{v}^T\underline{u}}{\underline{v}^T\underline{v}}\underline{v} - \underline{u} = 2\,\underline{v}\left(\frac{\underline{v}^T\underline{u}}{\underline{v}^T\underline{v}}\right) - \underline{u} =$$

dette er et tall

$$= \left(2\frac{\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}} - I\right)\underline{u} = Q_{\underline{v}}\,\underline{u}$$

5.3

$$Q_{\underline{v}}^T = \left(2\frac{\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}} - I\right)^T = 2\left(\frac{\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}}\right)^T - I^T = 2\frac{\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}} - I = Q_{\underline{v}}$$ dermed $Q_{\underline{v}}$ symmetrisk.

$$Q_{\underline{v}}^T Q_{\underline{v}} = \left(2\frac{\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}} - I\right)^2 = 4\left(\frac{\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}}\right)^2 - 4\frac{\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}} + I = 4\frac{\underline{v}\,\underline{v}^T\underline{v}\,\underline{v}^T}{(\underline{v}^T\underline{v})^2} - \frac{4\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}} + I =$$

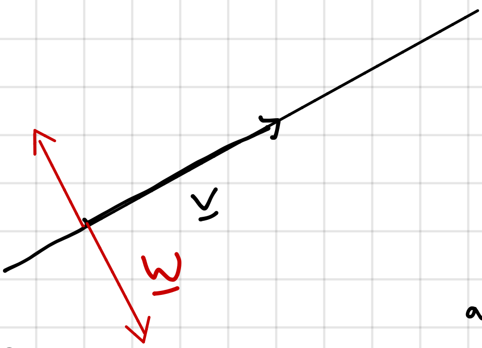
$$= 4\frac{\underline{v}\cancel{(\underline{v}^T\underline{v})}\underline{v}^T}{(\underline{v}^T\underline{v})^2} - \frac{4\underline{v}\,\underline{v}^T}{\underline{v}^T\underline{v}} + I = I$$ dermed $Q_{\underline{v}}$ ortogonal.

5.4

$Q_{\underline{v}}$ symmetrisk $\Rightarrow$ egenverdiene og egenvektorene er reelle.

i $\mathbb{R}^2$: man ser at refleksjonen av $\underline{v}$ med hensyn til $\underline{v}$ er $\underline{v}$ selv!

dvs $Q_{\underline{v}}\underline{v} = \underline{v} = 1 \cdot \underline{v}$ dermed $\underline{\underline{v}}$ er egenvektor av $Q_{\underline{v}}$ med egenverdi $\underline{\underline{\lambda = 1}}$




Hvis $\underline{\underline{\underline{w} \perp \underline{v}}}$, da ser vi at refleksjonen av $\underline{w}$ mht $\underline{v}$ er $-\underline{w}$

Dvs: $Q_{\underline{v}}\,\underline{w} = -\underline{w} = (-1)\,\underline{w}$ og $\underline{w}$ er egenvektor av $Q_{\underline{v}}$ med egenverdi $\underline{\underline{\lambda = -1}}$ ∎