UNIVERSITETET I OSLO
ØKONOMISK INSTITUTT

Eksamen i: **ECON3150/4150 – Elementær økonometri**
Exam: ECON3150/4150 – Introductory Econometrics

Eksamensdag: Onsdag 20. mai 2009	**Sensur kunngjøres: Fredag 12. juni 2009**
Date of exam: Wednesday, May 20, 2009	***Grades will be given: Friday, June 12, 2009***

Tid for eksamen: kl. 09:00 – 12:00
Time for exam: 09:00 a.m. – 12:00 noon

Oppgavesettet er på 8 sider
The problem set covers 8 pages ***English version on page 3***

Tillatte hjelpemidler:
- Alle trykte og skrevne hjelpemidler, samt kalkulator er tillatt

Resources allowed:
- *All written and printed resources, as well as calculator, is allowed*

Eksamen blir vurdert etter ECTS-skalaen. A-F, der A er beste karakter og E er dårligste ståkarakter. F er ikke bestått.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

I denne oppgaven skal vi studere etterspørselen etter bensin ved hjelp av data fra USA. Datasettet er kvartalsdata for perioden 1. kvartal 1959 til 4. kvartal 1990, og innholder variablene: $\tilde{Y}(t)$ - bensinutgifter per capita i faste priser, $\tilde{X}_1(t)$ - realpris per gallon bensin (en gallon er ca. 4,5 liter), $\tilde{X}_2(t)$ - realinntekt per capita og $\tilde{X}_3(t)$ - kjørelengde (målt i miles) per gallon. Våre analyser er basert på logaritmisk transformasjon av disse variable, dvs. $Y(t) = \ln \tilde{Y}(t)$, $X_1(t) = \ln \tilde{X}_1(t)$, $X_2(t) = \ln \tilde{X}_2(t)$ og endelig $X_3(t) = \ln \tilde{X}_3(t)$.

Vi tar utgangspunkt i regresjonen

(1) $$Y(t) = \beta_0 + \beta_1 X_1(t) + \beta_2 X_2(t) + \beta_3 X_3(t) + \varepsilon(t) \qquad t = 1,2,3,......,128$$

der restleddene $\varepsilon(t)$ antas å tilfredsstille betingelsene:
$$E(\varepsilon(t)) = 0, \quad Var(\varepsilon(t)) = \sigma^2, \quad Cov(\varepsilon(t), \varepsilon(h)) = 0 \quad for \quad t \neq h$$

Spørsmål 1
Hvordan vil du tolke regresjonskoeffisientene β_1 og β_2 i regresjonsligningen (1)?

Utskrift 1 viser resultatene når regresjon (1) anvendes på vårt datamateriale.

Spørsmål 2
Gi en kort og presis tolkning av resultatene slik de fremgår av Utskrift 1.

Resultatene fra Utskrift 1 kan tyde på at restleddene $\varepsilon(t)$ ikke tilfredsstiller standardbetingelsene slik de ble presisert ovenfor. For å undersøke dette spesifiseres $\varepsilon(t)$ som en stasjonær AR(1) prosess:

(2) $$\varepsilon(t) = \rho\varepsilon(t-1) + \nu(t) \qquad |\rho| < 1$$

der $\nu(t)$ er en helt tilfeldig prosess som oppfyller betingelsene:
$E(\nu(t)) = 0, \quad Var(\nu(t)) = \sigma_{\nu}^{2}, \quad Cov(\nu(t),\nu(h)) = 0 \quad t \neq h$.

Fra Utskrift 1 kan vi utlede de estimerte restleddene $\hat{\varepsilon}(t)$ der $\hat{\varepsilon}(t) = Y(t) - \hat{Y}(t)$. Utskrift 2 viser resultatene av regresjonen

(3) $$\hat{\varepsilon}(t) = \rho\hat{\varepsilon}(t-1) + u(t) \qquad t = 2,3,4,.........,128$$

Spørsmål 3
(a) Test nullhypotesen $H_0 : \rho = 0 \quad mot \quad alternativet \quad H_A : \ \rho > 0$ ved å bruke testen til Durbin-Watson med signifikansnivå $\alpha = 0.05$. (Du finner de relevante tabeller vedlagt)

(b) Test denne hypotesen når du tar utgangspunkt i estimatet $\hat{\rho}$ fra Utskrift 2, og du benytter samme signifikansnivå som ovenfor.

Ved store datasett (antall observasjoner er stort) er det vanlig å omgå problemet med autokorrelerte restledd ved å erstatte $\varepsilon(t)$ med $\hat{\varepsilon}(t)$ der $\hat{\varepsilon}(t)$ tilfredsstiller ligningen (3) ovenfor. Vi antar at denne approksimasjonen er tillatt for vårt datasett, slik at regresjon (1) kan reformuleres til

(4) $$Y(t) = \beta_0 + \beta_1 X_1(t) + \beta_2 X_2(t) + \beta_3 X_3(t) + \rho\hat{\varepsilon}(t-1) + u(t) \qquad t = 2,3,......,128$$

Resultatene for denne regresjonen er vist i Utskrift 3.

Spørsmål 4
Bruk opplysinger i Utskrift 3 til å teste nullhypotesen om at etterspørselen etter bensin har inntektselastisitet lik 1 mot den alternative hypotesen at den er forskjellig fra 1. Velg signifikansnivå $\alpha = 0.05$

Datasettet dekker oljekrisen høsten 73 og vinteren 74 som medførte økte bensinpriser. Antagelsen om at regresjonskoeffisientene i (4) er konstante for hele perioden kan derfor være tvilsom. For å undersøke dette har man estimert regresjonen (4) for periodene: fra 2. kvartal 1959 til og med 2. kvartal 1973, og fra 3. kvartal 1973 til 4. kvartal 1990. Resultatene av disse regresjonene er gitt i Utskrift 4 og i Utskrift 5.

For å ta hensyn til at regresjonskoeffisientene kan være forskjellig for de to del-periodene innfører vi dummyvariabelen

$$D(t) = \begin{cases} 1 \ \textit{for perioden } 2.\ k\text{var}\textit{tal } 1959 \ \textit{til } 2.\ k\text{var}\textit{tal } 1973 \\ 0 \ \textit{for perioden } 3.\ k\text{var}\textit{tal } 1973 \ \textit{til } 4.\ k\text{var}\textit{tal } 1990 \end{cases}$$

Deretter beregner vi regresjonen

$$\text{(5)} \qquad \begin{aligned} Y(t) &= \beta_0 + \gamma_0 D(t) + \beta_1 X_1(t) + \beta_2 X_2(t) + \beta_3 X_3(t) + \rho\hat{\varepsilon}(t-1) + \gamma_1 D(t) X_1(t) \\ &+ \gamma_2 D(t) X_2(t) + \gamma_3 D(t) X_3(t) + \gamma_4 D(t)\hat{\varepsilon}(t-1) + \delta(t) \qquad t = 2,3,4,.......,128 \end{aligned}$$

der $\delta(t)$ betegner stokastiske restledd.
I denne regresjonsligningen betegner variablene $D(t)X_j(t)$ produktet av $D(t)\ og\ X_j(t)$ $(j = 1,2,3),$ og $D(t)\hat{\varepsilon}(t-1)$ er produktet av $D(t)\ og\ \hat{\varepsilon}(t-1).$

Spørsmål 5
Den såkalte Chow-testen for at regresjonskoeffisientene i de to del-periodene er like er ekvivalent med å teste nullhypotesen
$H_0 : \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$ mot alternativet $H_A :$ minst en $\gamma_i \neq 0 \quad (i = 0,1,2,3,4)$
Still opp testobservatoren du vil benytte for denne testen og bruk opplysninger i de vedlagte utskrifter til å gjennomføre testen. Velg signifikansnivå $\alpha = 0.05$

Spørsmål 6
I Utskriften 6 finner vi at summen av de kvadrerte estimerte restledd

$$RSS = \sum_{t=2}^{128} \hat{\delta}(t)^2 = 0.0216878387$$

De tilsvarende summene for de kvadrerte estimerte restledd i regresjonene for de to del-periodene finner vi i Utskriftene 4 og 5 og er henholdsvis $0.0054274073\ \ og\ \ 0.0162604314$.

Vi ser at summen av disse blir 0.0216878387, dvs. nøyaktig lik summen $\sum_{t=2}^{128} \hat{\delta}(t)^2$ ovenfor.

Hvordan vil du forklare dette resultatet?

ENGLISH VERSION

In this exercise we shall study the demand for petrol using US quarterly data. The data set reports time series data for the period: from the first quarter of 1959 until the fourth quarter of 1990. The following variables are observed: $\tilde{Y}(t)$ - real petrol expenditure per capita, $\tilde{X}_1(t)$ - real price of petrol per gallon, $\tilde{X}_2(t)$ - real per capita income $\tilde{X}_3(t)$ - miles per gallon. Our analyses are based on logarithmic transformations of these variables, i.e. $Y(t) = \ln \tilde{Y}(t)$, $X_1(t) = \ln \tilde{X}_1(t)$, $X_2(t) = \ln \tilde{X}_2(t)$ and finally $X_3(t) = \ln \tilde{X}_3(t)$.

We start with the specification

$$\text{(1)} \qquad Y(t) = \beta_0 + \beta_1 X_1(t) + \beta_2 X_2(t) + \beta_3 X_3(t) + \varepsilon(t) \qquad t = 1,2,3,......,128$$

where the random disturbances $\varepsilon(t)$ are assumed to satisfy the conditions:
$E(\varepsilon(t)) = 0, \quad Var(\varepsilon(t)) = \sigma^2, \quad Cov(\varepsilon(t), \varepsilon(h)) = 0 \quad for \quad t \neq h$

Question 1
How will you interpret the coefficients β_1 and β_2 in regression (1)?

Output 1 shows the results when regression (1) is applied to our data.

Question 2
Give a short and concise interpretation of the results appearing in Output 1.

Results in Output 1 make us doubtful about the assumptions above regarding the random disturbances $\varepsilon(t)$. In order to examine the properties of the disturbances more closely, we specify $\varepsilon(t)$ as a stationary $AR(1)$ process

(2) $\qquad \varepsilon(t) = \rho\varepsilon(t-1) + \nu(t) \qquad |\rho| < 1$

where $\nu(t)$ is a purely random process satisfying the conditions:
$E(\nu(t)) = 0, \quad Var(\nu(t)) = \sigma_\nu^2, \quad Cov(\nu(t), \nu(h)) = 0 \quad t \neq h$.

Having run the regression (1) we derive the estimated disturbances $\hat{\varepsilon}(t)$ obtained from $\hat{\varepsilon}(t) = Y(t) - \hat{Y}(t)$. Output 2 shows the results of the regression:

(3) $\qquad \hat{\varepsilon}(t) = \rho\hat{\varepsilon}(t-1) + u(t) \qquad t = 2, 3, 4, ..., 128$

Question 3
(a) Test the null hypothesis $H_0 : \rho = 0 \quad against \quad the \quad alternative \quad H_A : \rho > 0$ by using the Durbin-Watson test with a level of significance $\alpha = 0.05$. (The relevant tables are enclosed)

(b) Test this hypothesis by utilizing the estimate $\hat{\rho}$ which you find in Output 2. Use the same level of significance as above.

When the number of observations are large it is customary to circumvent the problem of auto-correlated disturbances by replacing $\varepsilon(t)$ with $\hat{\varepsilon}(t)$ where $\hat{\varepsilon}(t)$ satisfies equation (3) above. We assume that this approximation is valid in our application, so that regression (1) can be reformulated to

(4) $\qquad Y(t) = \beta_0 + \beta_1 X_1(t) + \beta_2 X_2(t) + \beta_3 X_3(t) + \rho\hat{\varepsilon}(t-1) + u(t) \qquad t = 2, 3, ..., 128$

The results from this regression is shown in Output 3.

Question 4
Use the information you find in Output 3 to test the null hypothesis that the income elasticity of the demand for petrol is equal to 1 against the alternative hypothesis that it is different from 1. Choose level of significance $\alpha = 0.05$.

The so called oil crisis extending from autumn 73 till spring 74 is covered by our data set. This crisis caused large increases in the price of petrol. Our implicit assumption that the regression coefficients in (4) are constant for the whole sample period might therefore be doubtful. In order to investigate this issue we have run regression (4) for two sub-periods: we first applied regression (4) to the data covering the period from the second quarter of 1959 until the second quarter of 1973. The result of this regression is given in Output 4. Then we applied regression (4) to the remaining period, i.e. from the third quarter of 1973 until the fourth quarter of 1990. The results from this regression is shown in Output 5.

In order to take care of the fact that the regression coefficients can be different for the two sub-periods we introduce the dummy variable

$$D(t) = \begin{cases} 1 \text{ for the period: from the second quarter of } 1959 \text{ until the second quarter of } 1973 \\ 0 \text{ for the period: from the third quarter of } 1973 \text{ until the fourth quarter of } 1990 \end{cases}$$

Then we run the regression

$$(5) \qquad \begin{aligned} Y(t) &= \beta_0 + \gamma_0 D(t) + \beta_1 X_1(t) + \beta_2 X_2(t) + \beta_3 X_3(t) + \rho\hat{\varepsilon}(t-1) + \gamma_1 D(t) X_1(t) \\ &+ \gamma_2 D(t) X_2(t) + \gamma_3 D(t) X_3(t) + \gamma_4 D(t)\hat{\varepsilon}(t-1) + \delta(t) \qquad t = 2,3,4,.......,128 \end{aligned}$$

where $\delta(t)$ denotes the random disturbances.
In this regression equation the variables $D(t)X_j(t)$ denote the product of $D(t)$ *and* $X_j(t)$ for $(j = 1,2,3)$, and $D(t)\hat{\varepsilon}(t-1)$ is the product of $D(t)$ *and* $\hat{\varepsilon}(t-1)$.

Question 5
The so called Chow-test for testing whether the regression coefficients for the two sub-periods are equal is equivalent to testing the null hypothesis:
$H_0 : \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$ against the alternative H_A : at least one $\gamma_i \neq 0$ $(i = 0,1,2,3,4)$.
Define the test statistic you will use for this test. Then use information you find in the enclosed Outputs to carry out the testing procedure. Choose level of significance $\alpha = 0.05$.

Question 6
In Output 6 we can see that the sum of the squared residuals

$$RSS = \sum_{t=2}^{128} \hat{\delta}(t)^2 = 0.0216878387.$$

The corresponding sums of the squared residuals in the regressions for the two sub-periods are given in Output 4 and Output 5, and are, respectively, 0.0054274073 *and* 0.0162604314.
We observe that the sum of these numbers is 0.0216878387, i.e. equal to sum $\sum_{t=2}^{128} \hat{\delta}(t)^2$ calculated above..
How will you explain this result?

Utskrift 1/Output 1
EQ(1) Modelling Y by OLS (using tidsrekkedata.xls)
The estimation sample is: 1 to 128

	Coefficient	Std.Error	t-value	t-prob
Constant	-1.51454	0.1172	-12.9	0.000
X_1	-0.138561	0.01098	-12.6	0.000
X_2	0.998547	0.01540	64.8	0.000
X_3	-0.518128	0.01739	-29.8	0.000

sigma	0.0200694	RSS	0.0499449829
R^2	0.972774	F(3,124) =	1477 [0.000]**
		DW	= 0.741
no. of observations	128	no. of parameters	4
mean(Y)	-7.76303	var(Y)	0.014332

Utskrift 2/Output 2
EQ(2) Modelling $\hat{\varepsilon}$ by OLS (using tidsrekkedata.xls)
The estimation sample is: 2 to 128

	Coefficient	Std.Error	t-value	t-prob
$\hat{\varepsilon}_{-1}$	0.626791	0.06925	9.05	0.000

sigma	0.01545	RSS	0.0300763573
		DW	2.01
no. of observations	127	no. of parameters	1
mean($\varepsilon(t)$)	-0.000138973	var($\varepsilon(t)$)	0.000390795

Utskrift 3/Output 3
EQ(4) Modelling Y by OLS (using tidsrekkedata.xls)
The estimation sample is: 2 to 128

	Coefficient	Std.Error	t-value	t-prob
Constant	-1.52143	0.09277	-16.4	0.000
X_1	-0.136250	0.008594	-15.9	0.000
X_2	0.999642	0.01229	81.3	0.000
X_3	-0.517948	0.01367	-37.9	0.000
$\hat{\varepsilon}_{-1}$	0.627359	0.07038	8.91	0.000

sigma	0.0156941	RSS	0.0300490678
R^2	0.983027	F(4,122) =	1766 [0.000]**
		DW	2.01
no. of observations	127	no. of parameters	5
mean(Y)	-7.76104	var(Y)	0.01394

Utskrift 4/Output 4

EQ(5) Modelling Y by OLS (using tidsrekkedata.xls)
The estimation sample is: 2 to 58

	Coefficient	Std.Error	t-value	t-prob
Constant	0.852695	0.5579	1.53	0.132
X_1	-0.119027	0.06172	-1.93	0.059
X_2	0.798646	0.05333	15.0	0.000
X_3	-1.78133	0.3058	-5.82	0.000
$\hat{\varepsilon}_{-1}$	0.289868	0.1463	1.98	0.053

sigma	0.0102163	RSS	0.0054274073
R^2	0.994552	F(4,52) =	2373 [0.000]**
		DW	1.74
no. of observations	57	no. of parameters	5
mean(Y)	-7.84116	var(Y)	0.0174762

Utskrift 5/Output 5

EQ(6) Modelling Y by OLS (using tidsrekkedata.xls)
The estimation sample is: 59 to 128

	Coefficient	Std.Error	t-value	t-prob
Constant	-3.92774	0.6064	-6.48	0.000
X_1	-0.159297	0.01235	-12.9	0.000
X_2	0.519258	0.1218	4.26	0.000
X_3	-0.317965	0.05130	-6.20	0.000
$\hat{\varepsilon}_{-1}$	0.279561	0.1065	2.62	0.011

sigma	0.0158165	RSS	0.0162604314
R^2	0.852658	F(4,65) =	94.04 [0.000]**
		DW	1.82
no. of observations	70	no. of parameters	5
mean(Y)	-7.6958	var(Y)	0.00157655

Utskrift 6/Output 6

EQ(7) Modelling Y by OLS (using tidsrekkedata.xls)
The estimation sample is: 2 to 128

	Coefficient	Std.Error	t-value	t-prob
Constant	-3.92774	0.5220	-7.53	0.000
D	4.78043	0.9084	5.26	0.000
X_1	-0.159297	0.01063	-15.0	0.000
X_2	0.519258	0.1048	4.95	0.000
X_3	-0.317965	0.04416	-7.20	0.000
$\hat{\varepsilon}_{-1}$	0.279561	0.09170	3.05	0.003
DX_1	0.0402698	0.08294	0.48	0.628
DX_2	0.279389	0.1267	2.21	0.029

DX_3	-1.46336	0.4099	-3.57	0.001
$D\hat{\varepsilon}_{-1}$	0.0103071	0.2155	0.0478	0.962

sigma	0.0136149	RSS	0.0216878387
R^2	0.98775	F(9,117) =	1048 [0.000]**
		DW	1.83
no. of observations	127	no. of parameters	10
mean(Y)	-7.76104	var(Y)	0.01394

Table 5 **Critical Values for the Durbin–Watson Test: 5% Significance Level**[a]

	$K=2$		$K=3$		$K=4$		$K=5$		$K=6$		$K=7$		$K=8$		$K=9$		$K=10$		$K=11$	
T	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*	d_L^*	d_U^*
6	0.510	1.400																		
7	0.700	1.356	0.467	1.896																
8	0.763	1.332	0.559	1.777	0.368	2.287														
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588												
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414	0.243	2.822										
11	0.927	1.324	0.758	1.604	0.595	1.928	0.444	2.283	0.316	2.645	0.203	3.005								
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177	0.379	2.506	0.268	2.832	0.171	3.149						
13	1.010	1.340	0.861	1.562	0.715	1.816	0.574	2.094	0.445	2.390	0.328	2.692	0.230	2.985	0.147	3.266				
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030	0.505	2.296	0.389	2.572	0.286	2.848	0.200	3.111	0.127	3.360		
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977	0.562	2.220	0.447	2.472	0.343	2.727	0.251	2.979	0.175	3.216	0.111	3.438
16	1.106	1.371	0.982	1.539	0.857	1.728	0.734	1.935	0.615	2.157	0.502	2.388	0.398	2.624	0.304	2.860	0.222	3.090	0.155	3.304
17	1.133	1.381	1.015	1.536	0.897	1.710	0.779	1.900	0.664	2.104	0.554	2.318	0.451	2.537	0.356	2.757	0.272	2.975	0.198	3.184
18	1.158	1.391	1.046	1.5635	0.933	1.696	0.820	1.872	0.710	2.060	0.503	2.257	0.502	2.461	0.407	2.667	0.321	2.873	0.244	3.073
19	1.180	1.401	1.074	1.536	0.967	1.685	0.859	1.848	0.752	2.023	0.649	2.206	0.549	2.396	0.456	2.589	0.369	2.783	0.290	2.974
20	1.201	1.411	1.100	1.537	0.998	1.676	0.894	1.828	0.792	1.991	0.692	2.162	0.595	2.339	0.502	2.521	0.416	2.704	0.336	2.885
21	1.221	1.420	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964	0.732	2.124	0.637	2.290	0.547	2.460	0.461	2.633	0.380	2.806
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.940	0.769	2.090	0.677	2.246	0.588	2.407	0.504	2.571	0.424	2.734
23	1.257	1.437	1.168	1.543	1.078	1.660	0.986	1.785	0.895	1.920	0.804	2.061	0.715	2.208	0.628	2.360	0.545	2.514	0.465	2.670
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902	0.837	2.035	0.751	2.174	0.666	2.318	0.584	2.464	0.506	2.613
25	1.288	1.454	1.206	1.550	1.123	1.654	1.038	1.767	0.953	1.886	0.868	2.012	0.784	2.144	0.702	2.280	0.621	2.419	0.544	2.560
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873	0.897	1.992	0.816	2.117	0.735	2.246	0.657	2.379	0.581	2.513
27	1.316	1.469	1.240	1.556	1.162	1.651	1.084	1.753	1.004	1.861	0.925	1.974	0.845	2.093	0.767	2.216	0.691	2.342	0.616	2.470
28	1.328	1.476	1.255	1.560	1.181	1.650	1.104	1.747	1.028	1.850	0.951	1.958	0.874	2.071	0.798	2.188	0.723	2.309	0.650	2.431
29	1.341	1.483	1.270	1.563	1.198	1.650	1.124	1.743	1.050	1.841	0.975	19.44	0.900	2.052	0.826	2.164	0.753	2.278	0.682	2.396
30	1.352	1.489	1.284	1.567	1.214	1.650	1.143	1.739	1.071	1.833	0.998	1.931	0.926	2.034	0.854	2.141	0.782	2.251	0.712	2.363
31	1.363	1.496	1.297	1.570	1.229	1.650	1.160	1.735	1.090	1.825	1.020	1.920	0.950	2.018	0.879	2.120	0.810	2.226	0.741	2.333

[a] K refers to the number of columns in X, including the constant term.

Table 5 (*continued*)

T	$K=2$ d_L^*	$K=2$ d_U^*	$K=3$ d_L^*	$K=3$ d_U^*	$K=4$ d_L^*	$K=4$ d_U^*	$K=5$ d_L^*	$K=5$ d_U^*	$K=6$ d_L^*	$K=6$ d_U^*	$K=7$ d_L^*	$K=7$ d_U^*	$K=8$ d_L^*	$K=8$ d_U^*	$K=9$ d_L^*	$K=9$ d_U^*	$K=10$ d_L^*	$K=10$ d_U^*	$K=11$ d_L^*	$K=11$ d_U^*
32	1.373	1.502	1.309	1.574	1.244	1.650	1.177	1.732	1.109	1.819	1.041	1.909	0.972	2.004	0.904	2.102	0.836	2.203	0.769	2.306
33	1.383	1.508	1.321	1.577	1.258	1.651	1.193	1.730	1.127	1.813	1.061	1.900	0.994	1.991	0.927	2.085	0.861	2.181	0.795	2.281
34	1.393	1.514	1.333	1.580	1.271	1.652	1.208	1.728	1.144	1.808	1.080	1.891	1.015	1.979	0.950	2.069	0.885	2.162	0.821	2.257
35	1.402	1.519	1.343	1.584	1.283	1.653	1.222	1.726	1.160	1.803	1.097	1.884	1.034	1.967	0.971	2.054	0.908	2.144	0.845	2.236
36	1.411	1.525	1.354	1.587	1.295	1.654	1.236	1.724	1.175	1.799	1.114	1.877	1.053	1.957	0.991	2.041	0.930	2.127	0.868	2.216
37	1.419	1.530	1.364	1.590	1.307	1.655	1.249	1.723	1.190	1.795	1.313	1.870	1.071	1.948	1.011	2.029	0.951	2.112	0.891	2.198
38	1.427	1.535	1.373	1.594	1.318	1.656	1.261	1.722	1.204	1.792	1.146	1.864	1.088	1.939	1.029	2.017	0.970	2.098	0.912	2.180
39	1.435	1.540	1.382	1.597	1.328	1.658	1.273	1.722	1.218	1.789	1.161	1.859	1.104	1.932	1.047	2.007	0.990	2.085	0.932	2.164
40	1.442	1.544	1.391	1.600	1.338	1.659	1.285	1.721	1.230	1.786	1.175	1.854	1.120	1.924	1.064	1.997	1.008	2.072	0.945	2.149
45	1.475	1.566	1.430	1.615	1.383	1.666	1.336	1.720	1.287	1.776	1.238	1.835	1.189	1.895	1.139	1.958	1.089	2.022	1.038	2.088
50	1.503	1.585	1.462	1.628	1.421	1.674	1.378	1.721	1.335	1.771	1.291	1.822	1.246	1.875	1.201	1.930	1.156	1.986	1.110	2.044
55	1.528	1.601	1.490	1.641	1.452	1.681	1.414	1.724	1.374	1.768	1.334	1.814	1.294	1.861	1.253	1.909	1.212	1.959	1.170	2.010
60	1.549	1.616	1.514	1.642	1.480	1.689	1.444	1.727	1.408	1.767	1.372	1.808	1.335	1.850	1.298	1.894	1.260	1.939	1.222	1.984
65	1.567	1.629	1.536	1.662	1.503	1.696	1.471	1.731	1.438	1.767	1.404	1.805	1.370	1.843	1.336	1.882	1.301	1.923	1.266	1.964
70	1.583	1.641	1.554	1.672	1.525	1.703	1.494	1.735	1.464	1.768	1.433	1.802	1.401	1.837	1.369	1.873	1.337	1.910	1.305	1.948
75	1.598	1.652	1.571	1.680	1.543	1.709	1.515	1.739	1.487	1.770	1.458	1.801	1.428	1.834	1.399	1.867	1.369	1.901	1.339	1.935
80	1.611	1.662	1.586	1.688	1.560	1.715	1.534	1.743	1.507	1.772	1.480	1.801	1.453	1.831	1.425	1.861	1.397	1.893	1.369	1.925
85	1.624	1.671	1.600	1.696	1.575	1.721	1.550	1.747	1.525	1.774	1.500	1.801	1.474	1.829	1.448	1.857	1.422	1.886	1.396	1.925
90	1.635	1.679	1.612	1.703	1.589	1.726	1.566	1.751	1.542	1.776	1.518	1.801	1.494	1.827	1.469	1.854	1.445	1.881	1.420	1.909
95	1.645	1.687	1.623	1.709	1.602	1.732	1.579	1.755	1.557	1.778	1.535	1.802	1.512	1.827	1.489	1.852	1.465	1.877	1.442	1.903
100	1.654	1.694	1.634	1.715	1.613	1.736	1.592	1.758	1.571	1.780	1.550	1.803	1.528	1.826	1.506	1.850	1.484	1.874	1.462	1.898
150	1.720	1.746	1.706	1.760	1.693	1.774	1.679	1.788	1.665	1.802	1.651	1.817	1.637	1.832	1.622	1.847	1.608	1.862	1.594	1.877
200	1.758	1.778	1.748	1.789	1.738	1.799	1.728	1.810	1.718	1.820	1.707	1.831	1.697	1.841	1.686	1.852	1.675	1.863	1.65	1.874