UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON3150/ECON4150 – Introductory Econometrics**

Date of exam: Wednesday, May 15, 2013 **Grades are given: June 6, 2013**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 5 pages (incl. cover sheet)

Resources allowed:
- All written and printed resources, in addition to calculator, are allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Exam in: ECON 3150/4150: Introductory Econometrics

Day of exam: 15 May 2013

Time of day: 14:30-17:30

This is a 3 hour school exam.

Guidelines:
In the grading, question A will count 1/3 and question B will count 2/3.

Question A (1/3)

1. Consider the econometric model specified by (1)-(3)

   (1) $$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad i = 1, 2, ..., n$$

   (2) $$E(\varepsilon_i \mid X_i) = 0, \text{ for all } i$$

   (3) $$E(\varepsilon_i \varepsilon_j \mid X_i,\, X_j) = \begin{cases} \sigma^2, \text{ for } i = j \\ 0, \text{ for } i \neq j \end{cases}$$

   where the notation $\mid X_i$ means "conditional on any value that the variable X_i can take". Show that β_0 and β_1 can be interpreted as parameters of the conditional expectation function of Y_i given X_i.

2. Assume that the random variables $\{X_i, Y_i\}$ $(i = 1, 2, \ldots, n)$ have identical and independent joint probability density functions $f(x_i, y_i)$. How will you extend the model specification (1)-(3) if $f(x_i, y_i)$ $(i = 1, 2, \ldots, n)$ are binormal probability densities?

3. Let $\hat{\beta}_1$ denote the OLS estimator of the parameter β_1 in (1). Make use of the *Law of iterated expectations* to show that $E(\hat{\beta}_1) = \beta_1$ under the assumptions specified in (1)-(3).

4. Assume that a second model is formulated where $Var(\varepsilon_i)$ is proportional to the values of a random variable Z_i. In all other respects the second model is identical to (1)-(3). What is the expression for the BLUE estimator of β_1 in the second model?

5. Consider a third model for Y_i:

   (4) $$Y_i = \gamma_0 + \gamma_1 X_i + \gamma_2 Z_i + e_i, \quad i = 1, 2, ..., n$$

   (5) $$E(e_i \mid X_i, Z_i) = 0, \text{ for all } i$$

   (6) $$E(e_i e_j \mid X_i,\, Z_i, X_j, Z_j) = \begin{cases} \tau^2, \text{ for } i = j \\ 0, \text{ for } i \neq j \end{cases}$$

   (a) Express the probability limit *(plim)* of the OLS estimator $\hat{\beta}_1$ of model (1)-(3) in terms of parameters of the model (4)-(6).

(b) How is the precision of the OLS estimate of γ_1 affected by the degree of multi-collinearity? Explain briefly.

(c) Can we maintain that (1)-(3) is a valid regression model for Y_i, if the true model of Y_i given X_i and Z_i is (4)-(6) with $\gamma_2 \neq 0$? Explain briefly.

Question B (2/3)

In this problem, we model CPI inflation in Norway with the use of Phillips-curve models. We use annual data for the years 1981 to 2008. INF_t is the annual percentage change in the Consumer Price Index. U_t is the unemployment rate in percent.

When we estimate the static Phillips-curve model

$$INF_t = \beta_0 + \beta_1 U_t + \varepsilon_t$$

by OLS, we obtain the results

$$(7) \qquad \widehat{INF}_t = \overbrace{\underset{(1.59)}{8.69}}^{\hat{\beta}_0} + \overbrace{\underset{(0.465)}{(-1.42)}}^{\hat{\beta}_1} U_t$$

$$1981 - 2008 \ (T = 28), \ R^2 = 0.263, \ \hat{\sigma} = 2.86$$

$\widehat{INF}_t$ denotes the fitted values of inflation. Estimated standard errors are in round brackets below the estimates of the intercept and the slope coefficient. Below the estimated equation we have included information about the sample size (T), the coefficient of determination (R^2) and the estimated standard error of the disturbances ($\hat{\sigma}$).

1. If you use (7) to estimate the natural rate of unemployment, U^{nat}, the answer is $\hat{U}^{nat} = 6.12$ %. The estimated covariance between $\hat{\beta}_0$ and $\hat{\beta}_1$ in (7) is -0.70. Use the "delta-method" to show that an approximate standard error for the estimated natural rate is 1.01.

   Hint: Writing the non-linear estimator of the natural rate in the form: $\hat{U}^{nat} = \frac{\hat{\theta}_1}{\hat{\theta}_2}$, the delta-method formula for the standard error of $\hat{U}^{nat}$ can be expressed as:

   $$Var(\hat{U}^{nat}) = Var(\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right) \approx \left(\frac{1}{\hat{\theta}_2}\right)^2 \left[Var(\hat{\theta}_1) + \left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)^2 Var(\hat{\theta}_2) - 2\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right) Cov(\hat{\theta}_1, \hat{\theta}_2)\right]$$

2. Using the residuals $\hat{\varepsilon}_t$ ($t = 1982 - 2008$) from (7), we formulate the following auxiliary regression:

   $$(8) \qquad \hat{\varepsilon}_t = \underset{(0.085)}{0.764}\ \hat{\varepsilon}_{t-1} - \underset{(0.707)}{2.736} + \underset{(0.204)}{0.062}\ U_t$$

   $$1982 - 2008 \ (T = 27), \ R^2 = 0.778$$

   Use this result to test the null hypothesis of no first order autocorrelation in the disturbances of the static Phillips curve model.

3. What is the implication of the result of the autocorrelation test for the reliability of the inference that is based on the static Phillips-curve model (7)?

4. A near-to-hand extension of (7) is the following dynamic Phillips-curve model:

$$(9) \quad \begin{aligned} \widehat{INF}_t &= \underset{(0.985)}{2.736} + \underset{(0.080)}{0.75879}\, INF_{t-1} - \underset{(0.238)}{0.5998}\, U_t \\ & 1981 - 2008\ (T = 28),\ R^2 = 0.839,\ \hat{\sigma} = 1.361 \end{aligned}$$

   What is the intuitive explanation for why the estimated coefficient for U_t is markedly lower in absolute value in (9) than in (7)?

5. When we test for mis-specification of (9), there is no evidence of first order autocorrelation. Neither do the standard tests for heteroskedasticity and for departures from normality indicate model mis-specification. (We do not include the results of these tests in order to save notation and time). On this basis we will use (9) to test hypotheses about the properties of the Norwegian Phillips-curve.

   (a) Explain briefly why the OLS estimators of the regression coefficients in (9) are consistent (but biased in finite samples) under the assumption of no autocorrelation in the disturbances.

   (b) Test the null hypothesis that the long-run Norwegian Phillips-curve is vertical. What is the estimated slope of the long-run Phillips-curve?

   (c) When the sample period is extended to $1981 - 2012$, the residual sum of squares of the dynamic Phillips-curve is $RSS = 53.6490523$. When we also (in addition) include four indicator variables (dummies), one for each of the four new years in the sample, the residual sum of squares is $RSS = 46.3102852$. Test the hypothesis that there is no joint significance of these four indicator variables.

6. Imagine that you have a friend who is a business school student, and who has estimated a model for the change in inflation: $\Delta INF_t = INF_t - INF_{t-1}$. He has used exactly the same data for INF_t and U_t as you have used, and the sample period $(1981 - 2008)$. The equation he has estimated by OLS is:

$$(10) \quad \begin{aligned} \widehat{\Delta INF}_t &= \underset{(1.891)}{2.736} - \underset{(0.080)}{0.24121}\, INF_{t-1} - \underset{(0.238)}{0.5998}\, U_t \\ & 1981 - 2008\ (T = 28),\ R^2 = 0.314,\ \hat{\sigma} = 1.361 \end{aligned}$$

   Your friend cannot understand why he gets the same estimates for the intercept and for the coefficient of U_t, but a different estimate of the coefficient of INF_{t-1}. He is also worried that R^2 in (10) is lower than in your model (9). Can you resolve the puzzles for him? Explain briefly.

7. Your friend also suggests that you should try another estimation method than OLS. You agree that U_{t-1} may be a relevant instrumental variable for U_t, and decide to estimate the relationship between INF_t and U_t by the Methods

of Moments estimator, also called the Instrumental Variable estimator. You decide to re-estimate the static Phillips curve in (7). The results are

$$\widehat{INF}_t = \underset{(1.891)}{10.84} - \underset{(0.560)}{2.082}\ U_t \tag{11}$$
$$1981 - 2008\ (T = 28),\ \hat{\sigma} = 2.970$$

(there is no generally accepted "coefficient of determination" for the IV-estimator, which is why R^2 does not appear). The result for the regression between U_t and U_{t-1} is:

$$\widehat{U}_t = \underset{(0.316)}{0.6188} + \underset{(0.0941)}{0.8199}\ U_{t-1} \tag{12}$$
$$1981 - 2008\ (T = 28),\ R^2 = 0.731, \hat{\sigma} = 0.606$$

Compare the results in (11) with the results in (7) and comment in particular on the differences that you would expect to find based on your knowledge about the properties of the two estimators. Why is it relevant to test for instrumental variable strength, and what is the result in this case?