

Introduction to Econometrics- Econ 4150

Final Exam, Spring 2010

Suggested solutions

Problem 1(40 points)

- (i) $-53.533 + 30*1.014 + 40*1.024 = 17,847$ \$
- (ii) No. -53.333 applies (among others) to a person aged 0.
- (iii) Estimate of standard error of u is \$1000
- (iv) $\sqrt{3377.33364} = 58.1148$
- (v) F obs = 138.9, larger than a critical F value (df = 2 and 9267) at any reasonable level of significance. New model performs better.
- (vi) $1.427/(2*0.028) = 25.5$. It looks as if people under 25 are wealthier than those aged 25, but one should be extremely careful with such an out-of-sample prediction. Ever-increasing wealth on the age interval [25,64] is reasonable.
- (vii) The variables *inc* and *incsq* are strongly correlated (in fact is their correlation 0.94). Multicollinearity makes the coefficients of the two variables unstable.
- (viii) The two coefficients are not significant. They could just as well have been zero.

Problem 2 (30 points)

i) Substituting the equation for potential output in to the equation for current output it is possible to rewrite the model as

$$q_t = \lambda\gamma_0 + (1 - \lambda)q_{t-1} + \gamma_1\lambda p_t + \gamma_2\lambda x_{t-1} + u_t$$

ii) An OLS regression of q_t on q_{t-1}, p_t and x_{t-1} produces consistent, asymptotically normal estimators of the parameters. Under $E(e_t|p_t, q_{t-1}, p_{t-1}, x_{t-1}, \dots) = E(a_t|p_t, q_{t-1}, p_{t-1}, x_{t-1}, \dots) = 0$ it follows that $E(u_t|p_t, x_{t-1}, q_{t-1}, p_{t-1}, x_{t-2}, \dots) = 0$, which means that the model is dynamically complete [see equation (11.37)]. Therefore, the errors are serially uncorrelated. If the homoskedasticity assumption $Var(u_t|x_t, y_t) =$

σ_u^2 holds, then the usual standard errors, t statistics and F statistics are asymptotically valid.

iii) Because $\hat{\beta}_1 = (1 - \lambda)$ then $\hat{\beta}_1 = 0.5$ implies $\lambda = .5$. Further, $\hat{\beta}_2 = .2$ and $\hat{\beta}_3 = .2$, implies $\gamma_1 = \frac{\hat{\beta}_2}{\lambda} = \frac{.2}{.5} = .4$ and $\gamma_2 = \frac{\hat{\beta}_3}{\lambda} = \frac{.4}{.5} = .8$.

iv) No, because the error term is serially correlated AR(2) process which violates assumption TS.5'.

v) Given the assumptions u_t is an AR(2) process we could test the hypothesis

$$H_0 : \rho_0 = 0, \rho_1 = 0$$

Run the regression model and obtain \hat{u}_t and then use a F test for joint significance of \hat{u}_t and \hat{u}_{t-1} . If the test is significant at say 5%, we conclude that the errors are serially correlated.

vi) When $\rho_1 = 0$ then we have that the process is AR(1) and we use feasible GLS,

i) Run the OLS regression of q_t on q_{t-1}, p_t and x_{t-1} and obtain the residuals \hat{u}_t .

ii) Run the regression \hat{u}_t on \hat{u}_{t-1} for $t \geq 2$ and obtain $\hat{\rho}_0$.

iii) Apply OLS to the quasi-differenced data to estimate the parameters. The usual tests are asymptotically valid. Make sure you correct for the first time period.

Problem 3 (30 points)

(i) Disagree. Most time series processes are correlated over time, and many of them are strongly correlated. This means they cannot be independent across observations, which simply represent different time periods. Even series that do appear to be roughly uncorrelated – such as stock returns – do not appear to be independently distributed, under dynamic forms of heteroskedasticity.

(ii) Agree. This follows immediately from Theorem 10.1. In particular, we do not need the homoskedasticity and no serial correlation assumptions.

(iii) False. It is possible to construct an example of a model with a lagged depen-

dent variable and serially correlate errors where OLS is still consistent. For example

$$\begin{aligned}y_t &= \beta_0 + \beta_1 y_{t-1} + u_t \\ E(u_t | y_{t-1}) &= 0\end{aligned}$$

This model satisfies condition TS.3' and therefore β' s can be estimated consistently.

iv) True. We know that heteroskedasticity causes statistical inference based on the usual t and F statistics to be invalid, even in large samples.