UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: ECON3150/4150 – Introductory econometrics, spring 2004

Date of exam: Friday, August 13, 2004

Time for exam: 9:00 a.m. – 12:00 noon

The problem set covers 4 pages

Resources allowed:
- All written and printed resources, as well as calculator, are allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

In the cultivation of a plant called Jota, water is one of the input factors. Keeping the other input factors constant we are interested in clarifying the impact on the output (Y) by varying water input (X). In an experiment one applied different quantities of water. For each quantity of water applied (X_j) one observed the output (harvest) (Y_{ij}) for several plots of land. The data obtained from this experiment is shown in table 1 below.

In order to clarify the partial relation between the output (Y) and the input of water (X), we have run two regressions:

(1) $Y_{ij} = \beta_0 + \beta_1 X_j + \varepsilon_{ij} \qquad (i = 1,2..,n_j;\ j = 1,2,..,8)$.

(2) $Y_{ij} = \gamma_0 + \gamma_1 X_j + \gamma_2 X_j^2 + \delta_{ij} \quad (i = 1,2..,n_j;\ j = 1,2,..,8)$.

The results of these regressions applied to the present data are shown in Output 1 and Output 2.

Question 1

(a) Will you, from the empirical results as they appear in Output 1 and Output 2, conclude that there is a systematic relation between output Y and the input of water X ?

(b) Do you think the random disturbances in the two models satisfy the standard assumptions: i.e. independently, identically, normally distributed $(N(0,\sigma^2))$?

Question 2

Do you think it is justified to say that the expected marginal product of water ($\frac{\partial E(Y/X)}{\partial X}$) is constant over the domain of variation of X ? The notation $E(Y/X)$ means the expected value of Y for a given value of X.

Question 3

(a) Use model 1 and Output 1to calculate a prediction interval for (Y) when the amount of water applied is 15 inches and we choose 0.95 as the coefficient of confidence for the interval.

(b) From equation (1) it follows directly that $E(Y/X) = \beta_0 + \beta_1 X$. Calculate a prediction interval for $E(Y/X)$ when again the amount of water applied is 15 inches and the coefficient of confidence is 0.95. Explain the difference between the two intervals.

Output 3 shows the results of regression:

(3) $\overline{Y}_j = \alpha_0 + \alpha_1 X_j + \overline{\varepsilon}_j \qquad (j = 1,2,...,8)$

where $(\overline{Y}_j)$ denotes average output in each group and $(\overline{\varepsilon}_j)$ the average of the random disturbance terms in each group. We note that $\overline{\varepsilon}_j = (\sum_{i=1}^{n_j} \varepsilon_{ij})/n_j$ where n_j the number of observations in each group and $(j = 1,2,...,8)$. The observations on $\overline{Y}_j$ are given in the bottom line of Table 1.

Question 4

(a) Assume that the disturbances ε_{ij} are independently, identically $N(0, \sigma^2)$ distributed for all i and j. Use this information to deduce the properties of the disturbances $\overline{\varepsilon}_j$.

(b) Will OLS regression of $\overline{Y}_j$ on X_j provide us with BLUE estimators of α_0 and α_1 in this case?

Table 1

Quantity of water in inches (X_j)	0	12	18	24	30	36	48	60
Output in tons pr. Acre (Y_{ij})	2,35	4,31	5,69	6,00	7,35	7,58	8,05	5,55
	2,75	4,78	6,46	6,89	7,97	8,22	8,45	7,25
	2,89	4,84	7,02	7,69	8,32	8,63	8,63	10,17
	3,85	5,83	8,02	8,32	9,43	9,33	8,83	10,70
	5,52	6,51		8,38	9,54	9,38	9,52	
	5,94	7,52		9,96	11,06	12,48	10,62	
	3,88	5,63	6,80	7,92	8,98	9,27	9,02	8,42

Output 1:

EQ(1) Modelling Y by OLS. The estimation sample is: 1 to 44

	Coefficient	Std.Error	t-value	t-prob
Constant	5.02822	0.4747	10.6	0.000
X	0.0885630	0.01452	6.10	0.000

sigma	1.69559	RSS	120.750372
R^2	0.46961	F(1,42) =	37.19 [0.000]**
DW	1.28		
no. of observations	44	no. of parameters	2
mean(Y)	7.46773	var(Y)	5.17416

Normality test for Residuals

Observations	44
Mean	0.00000
Std.Devn.	1.6566
Skewness	-0.15812
Excess Kurtosis	0.93559

Asymptotic test Chi^2(2) = 1.7881 [0.40901]

Output 2:

EQ(2) Modelling Y by OLS The estimation sample is: 1 to 44

	Coefficient	Std.Error	t-value	t-prob
Constant	3.54599	0.5169	6.86	0.000
X	0.250858	0.03848	6.52	0.000
X^2	-0.00279656	0.0006295	-4.44	0.000

sigma	1.41002	RSS	81.514484
R^2	0.641951	F(2,41) =	36.75 [0.000]**
DW	1.74		
no. of observations	44	no. of parameters	3
mean(Y)	7.46773	var(Y)	5.17416

Normality test for Residuals

Observations	44
Mean	0.00000
Std.Devn.	1.3611
Skewness	0.45918
Excess Kurtosis	-0.15841

Asymptotic test: Chi^2(2) = 1.5922 [0.4511]

Output 3:

EQ(3) Modelling $\bar{Y}$ by OLS. The estimation sample is: 1 to 8

	Coefficient	Std.Error	t-value	t-prob
Constant	5.18932	0.7825	6.63	0.001
X	0.0807256	0.02314	3.49	0.013

sigma	1.19052	RSS	8.5040068
R^2	0.669709	F(1,6) =	12.17 [0.013]*
DW	0.682		
no. of observations	8	no. of parameters	2
mean($\bar{Y}$)	7.49	var$(\bar{Y})$	3.21838

Normality test for Residuals

Observations	8
Mean	0.00000
Std.Devn.	1.0310
Skewness	-0.21655
Excess Kurtosis	-1.2749
Asymptotic test:	Chi^2(2) = 0.60428 [0.73921]