Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 25 November 2024

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A gets 50 %, B 50 %.

Question A (50 %)

Consider the following equation for the stationary time series Y_t:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \beta_0 X_t + \beta_1 X_{t-1} + \epsilon_t, \tag{1}$$

where ϵ_t is gaussian white-noise and linearly uncorrelated with Y_{t-1}, Y_{t-2}, X_t, and X_{t-1}.

1. Show that equation (1) can be re-parameterized as:

$$\Delta Y_t = -\phi_2 \Delta Y_{t-1} + \beta_0 \Delta X_t + (\phi_1 + \phi_2 - 1)[Y_{t-1} - \frac{\phi_0}{1-\phi_1-\phi_2} - \frac{(\beta_0+\beta_1)}{1-\phi_1-\phi_2} X_{t-1}] + \epsilon_t, \tag{2}$$

   and explain why this is called an equilibrium correction model equation.

2. Assume that $\phi_1 = \phi_2 = 0.25$, $\beta_0 = 0.5$, and $\beta_1 = 0$. What are the first five dynamic multipliers (denoted δ_0, δ_1, δ_2, δ_3, δ_4) with respect to a one period increase in X_t?

3. In the rest of question A we simplify equation (1) by setting $\beta_0 = \beta_1 = 0$. Show that in this case equation (1) can be written as:

$$\Delta Y_t = -\phi_2 \Delta Y_{t-1} + (\phi_1 + \phi_2 - 1)[Y_{t-1} - E(Y_t)] + \epsilon_t. \tag{3}$$

4. In this question we simplify the model by setting $\phi_2 = 0$.

   (a) Assume that $\phi_0 = 1$ and $\phi_1 = 0.5$. Draw ("sketch") a figure that illustrates the conditional expectation function $E(Y_{T+h} \mid Y_T),\ h = 1, 2, 3, ..., H$ for the case where $Y_T = 3$.

   (b) Change the assumption about ϕ_1 to $\phi_1 = -0.5$. Draw a figure that illustrates the conditional expectation function for this case.

   (c) Assume that the period from $t = T$ to $t = T + H$ represents a forecast period, and that you use the conditional expectation to generate your dynamic forecasts, denoted $\hat{Y}_{T+h}$, $h = 1, 2, ..., H$. Under which assumptions will your forecasts be (MSE) optimal forecasts?

   (d) Assume that in period $t = T + 1$ there is a structural break in the form of an increase in $E(Y_t)$ due to a shift in ϕ_0 (we assume that ϕ_1 stays constant in the forecast period).

      i. Explain how this will cause a bias in the forecast of Y_{T+1} and Y_{T+2}.

      ii. Discuss how you can adjust the forecast for Y_{T+2} when the forecast can be made conditional on Y_{T+1}.

Question B (50 %)

In this question, we study the Norwegian nominal exchange rate, defined as kroner per unit of foreign currency. In Table 1 you find results for ADF-tests for the log of the nominal exchange rate, denoted LEX_t, and the log of the real exchange rate, denoted $LREX_t$. The definition of $LREX_t$ is:

$$LREX = LEX + LPK - LP, \tag{4}$$

where LPK_t is the log of the foreign consumer price index and LP_t is the log of the Norwegian consumer price index.

1. Explain how Table 1 can be used to test the null hypotheses that LEX_t is an I(1)-variable, and explain why the conclusion is that the null hypothesis is not rejected.

2. Explain why the results in Table 1 represent empirical evidence that does not support the hypothesis of relative Purchasing Power Parity (PPP).

3. Assume that LPK_t and LP_t are I(1)-series and consider the estimation results in Table 2.

   (a) Explain how the output can be used to test the hypothesis about no cointegration between LEX_t, LPK_t and LP_t, and give the result when the significance level is chosen to be 10 %.

   (b) What is the interpretation of the battery of mis-specification tests in the four last lines of the output and why are they relevant for the validity of the cointegration test? (Hint: Do not spend time on a detailed explanation of how the tests are construction, that is not the point here.)

4. Explain why the equation in Table 2 can be interpreted as a conditional model equation, obtained from a VAR with LEX_t, LPK_t and LP_t as endogenous variables and with second order dynamics.

5. Estimation of the VAR for LEX_t, LPK_t and LP_t gives the output for the Trace-test (Johansen-test) in Table 3. Use the output to test the hypotheses of cointegration rank between 0 and 2.

6. Explain the main differences between the ECM-test and the Trace-test of cointegration.

7. Based on $r = 1$, we can test the restrictions on the cointegration parameters that imply

$$\begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \times \begin{pmatrix} LEX_t \\ LPK_t \\ LP_t \end{pmatrix} \sim I(0).$$

   The test result is: $\chi^2(2) = 12.837[0.0016]$, where the value in parentheses is the p-value. What is the interpretation of the result for the hypothesis of relative purchasing power parity?

Tables with estimation results

Table 1: Dickey-Fuller tests of unit-root. LEX_t and LREX_t.

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Unit-root tests
The sample is: 2001(1) - 2024(2) (98 observations and 2 variables)

LEX:  ADF tests (T=94, Constant; 5%=-2.89 1%=-3.50)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 3     -0.8476      0.97418  0.02577   -0.1253   0.9006
 2     -0.9052      0.97331  0.02563    0.9717   0.3338
 1     -0.6975      0.98001  0.02562    0.8629   0.3905
 0     -0.5315      0.98512  0.02558

LREX: ADF tests (T=94, Constant; 5%=-2.89 1%=-3.50)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
  3      -1.007      0.96369  0.02526   -0.4490   0.6545
  2      -1.172      0.95942  0.02515    0.9494   0.3450
  1     -0.9570      0.96803  0.02513    1.027    0.3070
  0     -0.7404      0.97592  0.02514
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Table 2: Estimation results for LEX equation.

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EQ(1) Modelling DLEX by OLS

The estimation sample is: 2001(1) - 2024(2)

                  Coefficient  Std.Error  t-value  t-prob
DLEX_1               0.206117    0.09656     2.13  0.0357
DLPK                 -3.71246     0.6371    -5.83  0.0000
DLPK_1                2.88516     0.6324     4.56  0.0000
DLP                   1.06003     0.3900     2.72  0.0080
DLP_1               -0.612154     0.3746    -1.63  0.1059
LEX_1               -0.204627    0.05829    -3.51  0.0007
LPK_1               -0.452412     0.2007    -2.25  0.0268
LP_1                 0.549849     0.2120     2.59  0.0112
Constant            0.0165641   0.007232     2.29  0.0245

sigma               0.0208094  RSS             0.0368075901
R^2                  0.390514  F(8,85) =      6.808 [0.000]**
Adj.R^2              0.333151  log-likelihood        235.351
no. of observations        94  no. of parameters           9
mean(DLEX)         0.00154177  se(DLEX)            0.0254827

AR 1-5 test:      F(5,80)   =  0.71601 [0.6132]
ARCH 1-4 test:    F(4,86)   =  0.32068 [0.8634]
Normality test:   Chi^2(2)  =  0.46178 [0.7938]
Hetero test:      F(16,77)  =   1.3217 [0.2062]
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Table 3: Cointegration test results based on VAR(2) for LEX_t, LPK_t and LP_t.

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I(1) cointegration analysis, 2001(1) - 2024(2)

 eigenvalue    loglik for rank
                 959.6304   0
   0.19507       969.8292   1
  0.098003       974.6770   2
  0.042158       976.7014   3

 H0:rank<=  Trace test [ Prob]
     0          34.142 [0.014] *
     1          13.744 [0.090]
     2          4.0488 [0.144]

Asymptotic p-values based on: Unrestricted constant
Unrestricted variables:
[0] = Constant
Number of lags used in the analysis: 2

beta (scaled on diagonal; cointegrating vectors in columns)
LEX          1.0000    -0.035918      -0.10499
LP          -3.2193       1.0000      -0.98958
LPK          2.8778      -1.3449        1.0000

alpha
LEX        -0.14369     -0.10251       0.11610
LP         0.033787    -0.036559     -0.024958
LPK      -0.0067060   -0.0041676     -0.030521
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