Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 25 November 2025

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A gets 50 %, B 50 %.

1 Question A (50 %)

Consider the static data generating process (DGP) of Y_t and X_t:

$$\begin{pmatrix} Y_t \\ X_t \end{pmatrix} \sim IIN(\mu, \Sigma) \tag{1}$$

with expectation:

$$\mu = \begin{pmatrix} 1 \\ 0.1 \end{pmatrix} \tag{2}$$

and variance-covariance matrix:

$$\Sigma = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}. \tag{3}$$

1. Explain why the DGP implies the conditional expectation:

$$E(Y_t \mid X_t) = 0.95 + 0.5X_t \tag{4}$$

and the conditional variance:

$$Var(Y_t \mid X_t) = 0.75. \tag{5}$$

2. Consider the dynamic DGP of Y_t and X_t:

$$\begin{pmatrix} Y_t \\ X_t \end{pmatrix} \Bigg| Y_{t-1}, X_{t-1} \sim IIN(\mu_t, \Sigma \mid Y_{t-1}, X_{t-1}) \tag{6}$$

where the conditional expectations are:

$$\mu_t = \begin{pmatrix} \mu_{Yt} \\ \mu_{Xt} \end{pmatrix} = \begin{pmatrix} 1 + 0.9Y_{t-1} \\ 0.1 + 0.8X_{t-1} \end{pmatrix} \tag{7}$$

and the conditinal covariance matrix Σ has the same elements as in (3).
Explain why this DGP can be written as a VAR where there is no mutual Granger-causality.

3. Show that the conditional model equation of Y_t consistent with the DGP in Question A2 is:

$$Y_t = 0.95 + 0.5X_t + 0.9Y_{t-1} - 0.4X_{t-1} + \epsilon_{Yt}, \tag{8}$$

and explain why the marginal model equation of X_t is:

$$X_t = 0.1 + 0.8X_{t-1} + \varepsilon_{Xt}. \tag{9}$$

4. Explain why the time series generated by (8) and (9) are stationary.

5. Show that the unconditional expectations are $E(Y_t) = 10$ and $E(X_t) = 0.5$.

6. Calculate the three first dynamic multipliers implied by equation (8).

7. Show that (8) implies that the long-run multiplier of Y with respect to X is $+1$.

8. Assume that the model (8) and (9) is used to forecast $Y_{T+1}, Y_{T+2}, ..., Y_{T+H}$ conditional on Y_T and X_T.

   (a) Use the conditional expectation function $E(Y_{T+h} \mid Y_T, X_T)$,$h = 1, 2, ..H$ to make forecasts. Assume that $Y_T = 5$ and $X_T = 0$. What are the (point) forecasts of Y_{T+1} and Y_{T+2} ?

   (b) What is the forecast of Y_{T+H} when H grows towards infinity, i.e., what is the so called long-term forecast?

Question B (50 %)

In this question you are asked to interpret and comment several empirical results obtained from modelling annual time series data for the Norwegian Consumer Price Index (CPI) of food. The following variable names are used:

LCPI:	Natural logarithm of the food consumer price index.
LFHP:	Natural logarithm of the price index of first hand sales of food products.
LEP:	Natural logarithm of the index of the system price of electricity.
First differences are denoted by DLCPI, DLFHP and DLEP.	

1. Table 1 shows results of unit-root tests of LCPI and LFHP and their first differences. Explain how you can use the information to defend the decision that both LCPI and LFHP are integrated of order one, $I(1)$.

2. Table 2 shows estimation results for a conditional model equation of the food price index with DLCPI as the dependent variable.

   (a) Give brief explanations of the "battery" of residual mis-specification tests. Assume that a 1 % significance level can be applied for the assessment of the significance of the tests.

   (b) The R^2 of the equation is seen to be 0.707698. Assume that a business school student tells you that it is better to regress LCPI on the explanatory variables in Table 2 because the R^2 of that model equation is 0.995302. What would be your refutation of the argument?

   (c) Explain how the estimation results shown in Table 2 can be used to test cointegration between LCPI and LFHP. Use a significance level of 5 % for the testing of the null-hypothesis of no cointegration.

   (d) Conditional on cointegration, show that the error correction term can be written as:

   $$\text{ECMcpi}_t = LCPI_t - 0.517LFHP_t - 0.038 \qquad (10)$$

   and explain why the variable can be interpreted as an I(0) time series.

3. Table 3 shows the estimated conditional model equation, with lagged ECMcpi_t as explanatory variable. What is the food price change predicted by this model in the special case of $\text{DLFHP}_t = 0$ and $\text{ECMcpi}_{t-1} = 0$? Is the change statistically significant?

4. Assume that it is suggested that you check the robustness of the estimation results by using Instrumental Variable Estimation (IVE). Such an estimation is shown in Table 4. What are your comments to the results, and to the "Specification test" in particular?

5. Assume that a business school student says that you should use 2SLS instead of IVE. What would your answer be?

6. Assume that a fellow students of econometrics asks you how you can test the hypothesis that the first-hand sales price index is weakly exogenous with respect to the cointegration parameters. Explain how the results in Table 5 do not reject that exogeneity hypothesis.

7. When the conditional model equation of DLCPI is estimated on the sample 1982-2024, the Residual Sum of Squares becomes RSS= 0.0142466471. Use this information together with RSS in Table 3 to show that a Chow-type test of parameter constancy for period 2020-2024 becomes:
   $F(5,35) = 2.4813$ [0.0503]
   (Some rounding errors will be looked mildly upon here.)
   Interpret the result. Is there other tests or graphs that would have been useful for assessment of the constancy of the parameters, or of lack thereof?

8. The Johansen method allows inference about cointegration without first making any assumptions about weak exogeneity. Table 6 contains results of a cointegration analysis by the use of the Johansen method. In this analysis, the VAR included two lags of LCPI and LFHP, hence VAR(2), and the sample used was 1981-2019. Explain how you interpret the results.

Tables with estimation results

Table 1: Dickey-Fuller tests of unit-root.

Unit-root tests

The sample is: 1982 - 2019 (41 observations and 2 variables)

LCPI: ADF tests (T=38, Constant; 5%=-2.94 1%=-3.61)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-2.806	0.95298	0.01621	-1.982	0.0552
1	-2.091	0.96599	0.01697	4.294	0.0001
0	-6.941**	0.91221	0.02067		

LFHP: ADF tests (T=38, Constant; 5%=-2.94 1%=-3.61)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-0.6491	0.99082	0.02067	0.8281	0.4134
1	-0.8550	0.98825	0.02058	4.360	0.0001
0	-2.290	0.96460	0.02521		

Unit-root tests

The sample is: 1983 - 2019 (39 observations and 2 variables)

DLCPI: ADF tests (T=37, Constant; 5%=-2.94 1%=-3.62)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
1	-3.992**	0.62034	0.01794	0.6538	0.5177
0	-4.057**	0.61769	0.01779		

DLFHP: ADF tests (T=37, Constant; 5%=-2.94 1%=-3.62)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
1	-2.787	0.62925	0.02069	-0.9801	0.3339
0	-3.146*	0.59577	0.02068		

Table 2: Conditional model equation of food-CPI.

```
Modelling DLCPI by OLS
The estimation sample is: 1982 - 2019

                   Coefficient  Std.Error  t-value  t-prob
LCPI_1               -0.142609    0.04137    -3.45  0.0015
LFHP_1               0.0737266    0.04002     1.84  0.0742
Constant            0.00536807    0.01236    0.434  0.6667
DLFHP                 0.369276     0.1241     2.98  0.0054

sigma                0.0175876  RSS             0.0105170283
R^2                   0.707698  F(3,34) =       27.44 [0.000]**
Adj.R^2               0.681907  log-likelihood        101.735
no. of observations         38  no. of parameters           4
mean(DLCPI)          0.0286143  se(DLCPI)           0.0311839

AR 1-2 test:      F(2,32)    =   4.4331 [0.0200]*
ARCH 1-1 test:    F(1,36)    =  0.31301 [0.5793]
Normality test:   Chi^2(2)   =   2.7837 [0.2486]
Hetero test:      F(6,31)    =   1.7414 [0.1445]
RESET23 test:     F(2,32)    =   1.0719 [0.3543]
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Table 3: Conditional model equation of food-CPI, conditional on cointegration.

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Modelling DLCPI by OLS
The estimation sample is: 1982 - 2019

                   Coefficient  Std.Error  t-value  t-prob
ECMcpi_1             -0.144770    0.02333    -6.21  0.0000
DLFHP                 0.368525     0.1218     3.03  0.0046
Constant           0.000605670   0.004334    0.140  0.8897

sigma                0.0173356  RSS             0.0105182833
R^2                   0.707663  F(2,35) =       42.36 [0.000]**
Adj.R^2               0.690958  log-likelihood        101.733
no. of observations         38  no. of parameters           3
mean(DLCPI)          0.0286143  se(DLCPI)           0.0311839
```

Table 4: IV-estimated equation of food-CPI, conditional on cointegration.

```
Modelling DLCPI by IVE
The estimation sample is: 1982 - 2019

                   Coefficient  Std.Error  t-value  t-prob
DLFHP          Y      0.551963     0.2068     2.67  0.0114
ECMcpi_1             -0.128046    0.02835    -4.52  0.0001
Constant           -0.00271187   0.005369   -0.505  0.6167

sigma                0.0178888  RSS             0.0112003259
Reduced-form sigma    0.017308
no. endogenous variables     2  no. of instruments          5
no. of observations         38  no. of parameters           3
mean(DLCPI)          0.0286143  se(DLCPI)           0.0311839
Additional instruments:
[0] = DLFHP_1
[1] = DLEP
[2] = DLEP_1

Specification test: Chi^2(2)  =   3.7521 [0.1532]
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Table 5: Estimation results for food first hand sales price index, conditional on cointegration.

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   Modelling DLFHP by OLS
  The estimation sample is: 1982 - 2019

                 Coefficient  Std.Error  t-value  t-prob
DLFHP_1             0.491375     0.1387     3.54  0.0011
ECMcpi_1          -0.0221191    0.03126   -0.708  0.4838
Constant           0.0105370   0.004930     2.14  0.0396

sigma              0.0206452  RSS             0.0149178487
R^2                 0.430649  F(2,35) =      13.24 [0.000]**
Adj.R^2             0.398115  log-likelihood       95.0932
no. of observations       38  no. of parameters          3
mean(DLFHP)        0.0289945  se(DLFHP)          0.0266111

AR 1-2 test:      F(2,33)   = 0.021783 [0.9785]
ARCH 1-1 test:    F(1,36)   = 0.096000 [0.7585]
Normality test:   Chi^2(2)  =  0.20437 [0.9029]
Hetero test:      F(4,33)   =  0.42943 [0.7863]
RESET23 test:     F(2,33)   =  0.59628 [0.5567]
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Table 6: Results of Johansen method of cointegration analysis. PART A shows the results of testing of cointegration rank. PART B shows results based on the decision about cointegration rank.

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PART A: TESTING COINTEGRATION RANK (NUMBER OF COINGRATION RELATIONSHIPS)

I(1) cointegration analysis of VAR(2). Sample 1981 - 2019

 eigenvalue    loglik for rank
                192.5516   0
    0.35578     201.1262   1
  0.0033450     201.1915   2

 H0:rank<=  Trace test [ Prob]
     0          17.280 [0.025] *
     1         0.13067 [0.718]

Asymptotic p-values based on: Unrestricted constant
Unrestricted variables:
[0] = Constant

beta (scaled on diagonal; cointegrating vectors in columns)
LCPI         1.0000     -0.88507
LFHP       -0.51939       1.0000

alpha
LCPI       -0.13569   -0.0015211
LFHP      -0.053079    -0.016606

PART B: COINTEGRATED VAR(2).
Sample 1981 - 2019.

Cointegrated VAR in:
[0] = LCPI
[1] = LFHP
Unrestricted variables: Constant

beta
LCPI         1.0000
LFHP       -0.51939

alpha
LCPI       -0.13569
LFHP      -0.053079

Standard errors of alpha
LCPI       0.031023
LFHP       0.036537
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