Exam in ECON3220/4220, Fall 2020

Problem 1 (*Microeconomics*)

November 25, 2020

Part A: consumer theory (weight 14%, equally shared)

1. For each of the axioms on preferences in Chapter 1 (completeness, transitivity, continuity, strict monotonicity, and strict convexity), represent graphically preferences that do not satisfy that axiom.
2. Find the indirect utility function associated to the utility function $U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$ with $\alpha, \beta > 0$ and check Roy's identity for good 2.

Part B: production theory (weight 15%, equally shared)

1. Explain the economic meaning of concavity with respect to prices of the cost function.
2. For each of the following, show that these cannot be cost functions (these violate some of its properties):
   a) $c(\boldsymbol{w}, y) = y^2 (w_1 + e^{w_2})$;
   b) $c(\boldsymbol{w}, y) = \sqrt{(1-y) w_1 w_2}$;
   c) $c(\boldsymbol{w}, y) = y \left(\frac{w_1}{w_2}\right)$;
3. For the cost function $c(\boldsymbol{w}, y) = y^2 w_1$, derive the production function and the profit function.

Part C: general equilibrium and taxation (weight 21%, equally shared)

Corn is used for both the production of food and for the production of ethanol, an alternative fuel. Denote the quantities of corn by x_C, food by x_F, and ethanol by x_E.

There are two representative firms, operating respectively in the food and ethanol sectors. The production functions in both the food industry and the ethanol sectors are constant returns to scale. Let z_C denote the input of corn, y_F denote the output of food, and y_E the output of ethanol. Then, $y_F \leq f_F(z_C) = Az_C$ with $A > 0$ and $y_E \leq f_E(z_C) = z_C$.

The representative individual has a utility function $U(x_F, x_E) = \ln x_F + x_E$ and an endowment of corn ω_C.

The market is perfectly competitive.

1. Verify that the conditions of the first welfare theorem are satisfied.

2. Compute the general equilibrium outcome. [*Hint: the easiest method is to directly identify the optimal quantities, without using prices, demand, supply, etc.*]

3. Briefly explain the inverse elasticity rule for optimal commodity taxation and discuss its implications for the taxation of food and ethanol. *[Clarification: no need to find the optimal taxes nor the elasticities!]*

Exam in ECON3220/4220, Fall 2020

Problem 2 (Game theory and the economics of information)

Part A: Rationalizable strategies and mixed-strategy Nash equilibrium

Weight: 16% *(with equal weight* = 8% *on each subproblem)*

Consider the following normal form game.

	L	C	R
U	0, 0	2, 0	1, 4
M	−1, 4	3, 1	3, 2
D	2, 0	2, 3	0, 1

(a) What strategies are rationalizable for each of the two players?

(b) The game has one and only one Nash equilibrium, in mixed strategies. Find the mixed strategies that the players use in this Nash equilibrium.

Part B: Auctions under complete and incomplete information

Weight: 24% *(with equal weight* = 8% *on each subproblem)*

Consider a private value second-price sealed-bid auction with two bidders who both are risk-neutral, have valuations in the interval $[0, 1]$ and can submit bids in the interval $[0, 1]$. In a private value second-price sealed-bid auction the bidders are first informed of their valuations, they then submit their bids, the item is assigned to the bidder with the highest bid (and with equal probability to each if the two bids are equal), and the winner pays the second highest bid (i.e., the bid of the other bidder when there are only two bidders). The utility of the winner equals his/her valuation minus the payment, while the other bidder has zero utility.

(a) Assume first that the valuations of the two player are commonly known, with $v_1 = 1$ and $v_2 = 0$. One Nash equilibrium of the auction is that the players bid their valuations ($b_1 = 1$ and $b_2 = 0$), as bidder 1's utility equals 1 for any positive bid and 1 with probability equal to 0.5 for a bid equal to 0, and bidder 2's utility equals 0 for any bid less than 1 and -1 with probability equal to 0.5 for a bid equal to 1.

Another Nash equilibrium is that $b_1 = 0$ and $b_2 = 1$. Why?

(b) Assume now that the valuations are independent and identically distributed on the interval $[0, 1]$, and each bidder is only informed of his/her own valuation. Consider that each player $i = 1, 2$ bids his/her valuation, i.e., $b_1(v_1) = v_1$ for all v_1 in $[0, 1]$ and $b_2(v_2) = v_2$ for all v_2 in $[0, 1]$. Why is this a Bayesian Nash equilibrium?

(c) Assume still, as in part (b), that the valuations are independent and identically distributed on the interval $[0, 1]$, and each bidder is only informed of his/her own valuation. But consider now that bidder 1 always bids 0, independently of his/her valuation, i.e., $b_1(v_1) = 0$ for all v_1 in $[0, 1]$, and that bidder 2 always bids 1, independently of his/her valuation, i.e., $b_2(v_2) = 1$ for all v_2 in $[0, 1]$. Why is this also a Bayesian Nash equilibrium?

Part C: Perfect Bayesian equilibrium

Weight: 10%

Consider the following extensive form game, where the first number is the payoff of player 1 and the second number is the payoff of player 2. Show that this game has one and only one Perfect Bayesian equilibrium.