Problem 1 (Microeconomic theory). Weights are as specified.

Part A (20%):

Andrea has the following indirect utility function:

$$V(p_1, p_2, m) = \frac{m}{p_1 + p_2}.$$

A1 (4%): Derive Andrea's Marshallian demands of the goods x_1 and x_2.

A2 (4%): Briefly discuss the complementarity/substitutability of the goods.

A3 (4%): Use the relationship between indirect utility and expenditure function to find the expenditure function and, then, the Hicksian demands.

A4 (4%): Draw the indifference curves of Andrea and discuss its properties.

A5 (4%): Discuss the effect of taxation of good 1 for Andrea.

Part B (15%):

A firm (in a standard price taking setting) "Nut" produces with the technology:

$$y \leq f_n(z_1, z_2) = (z_1 z_2)^{\frac{1}{4}}.$$

B1 (8%): Derive the conditional demand for inputs and the cost function of Nut.

B2 (7%):Find the profit function for this firm.

Part C (15%):

Consider an economy where the only agents are:

(Andrea's cousin) Carl, with preferences $U(x_1, x_2) = \min[x_1, x_2]$ and an endowment vector $(\omega_1, \omega_2) = (0, 2)$; and

the firm "Bolt" that produces with the technology:

$$y \leq f_b(z_1) = z_1^2.$$

Good 1 (x_1) is the same as the output y of the firm. Good 2 (x_2) is the same as the input z_1 of the firm.

C1 (4%): Discuss the profit maximization problem of the firm Bolt.

C2 (4%): Determine the set of feasible allocations for this economy.

C3 (4%): Argue that the allocation of consumption $(x_1^*, x_2^*) = (1, 1)$ and of production $(y^*, z_1^*) = (1, 1)$ is Pareto efficient.

C4 (3%): Can this allocation be a Walrasian equilibrium allocation? Briefly justify your answer (Hint. You can use two arguments: one based on the profit maximization problem of the firm; and one based on the assumptions of the second welfare theorem).

Exam in ECON3220/4220, Fall 2021

Problem 2 (Game theory and the economics of information)

Part A: Nash and Bayesian Nash equilibrium

Weight: 30% *(with equal weight* = 3% *on each subproblem)*

Consider a simultaneous move duopoly game with homogeneous goods in which two firms 1 and 2 compete by selecting quantities q_1 and q_2 respectively and where the price is determined by the inverse demand function given by: $p = 60 - q_1 - q_2$. Firm 1's cost is 18 per unit of production. Firm 2's cost is c_2 per unit of production. There are no fixed costs and each firm maximizes its own profits.

(a) Write down the profit function of each firm.

(b) Without reference to this specific problem, explain what a best response function is, what a Nash Equilibrium is, and what a rationalizable strategy is.

(c) The best response function for each of the firms is given by

$$q_1 = \frac{60 - q_2 - 18}{2} \qquad \text{and} \qquad q_2 = \frac{60 - q_1 - c_2}{2}\,.$$

Show how each of these functions are derived from the corresponding profit function.

Assume for subproblems (d), (e), and (f) that $c_2 = 18$ and that this is commonly known among the firms.

(d) What quantities for each of the firms can be best responses to some belief about the quantity of the other firm.

(e) Solve for the Nash Equilibrium.

(f) What quantities for each of the firms are rationalizable?

Assume for subproblems (g), (h), (i), and (j) that firm 1 is unsure about c_2 but believes with probability $\frac{1}{2}$ that $c_2 = 12$ and with probability $\frac{1}{2}$ that $c_2 = 24$. Assume furthermore that firm 2 knows whether $c_2 = 12$ or $c_2 = 24$ and also knows firm 1's beliefs.

(g) Specify the profit function for firm 1, and calculate its best response function under the assumption that firm 1 is risk-neutral and thus maximizes expected profit.

(h) Specify the best response function for each type of firm 2.

(i) Explain in words what a Bayesian Nash Equilibrium is in this game.

(j) Solve for the Bayesian Nash Equilibrium.

Part B: Adverse section

Weight: 20% *(with equal weight* = 5% *on each subproblem)*

(a) Consider the model of an insurance market presented in Section 8.1.3 in Jehle and Reny (2011). Describe the pure strategy subgame-perfect equilibrium in this model, if such an equilibrium exist.

(b) In this model, why is the equilibrium necessarily separating, if it exists? Why does not such an equilibrium exist if the fraction α of low-risk consumers is sufficiently large?

(c) The model of Section 8.1.3 in Jehle and Reny (2011) is presented in the context of motor insurance, but it provides insights also for health insurance. Assume that health insurance companies are only allowed to offer contracts that fully insure the consumers against the cost of treatment. Consumers' probability of falling ill is private information, and it is either low or high. What two equilibrium outcomes can arise?

(d) The Affordable Care Act (Obamacare) in the United States (i) requires that essential health benefits must be provided and (ii) imposes an individual mandate requiring everyone to have insurance or pay a penalty. In context of the model described in Section 8.1.3 in Jehle and Reny (2011), explain the rationale behind these two provisions of the Affordable Care Act.