nhf, 03.11.25

ECON 3220-4220

Exam

Question 1 (10%)

Define and explain the following terms: risk, risk aversion, beliefs and Perfect Bayesian equilibrium.

Question 2 (30%)

An entrepreneur has developed a potential project. If the entrepreneur invests an amount $x \geq 0$ in the project it gives a return $\overline{r}x$ with probability π and a return $\underline{r}x$ with probability $1-\pi$, where $\underline{r} < 0 < \overline{r}$.

A

Assume that the entrepreneur values monetary gains y according to a twice continuously differentiable, increasing and concave utility function $v(y)$, with $v'(y) > 0$ and $v''(y) < 0$. Explain why this implies that the entrepreneur is risk averse and that the entrepreneur therefore prefers the expected value of the return on the investment, $v\left(\left(\pi\overline{r} + (1-\pi)\underline{r}\right)x\right)$, to the expected utility of the project, $\pi v(\overline{r}x) + (1-\pi)v\left(\underline{r}x\right)$. Illustrate your argument in a figure.

B

Define $Eu(x) = \pi v(\overline{r}x) + (1-\pi)v\left(\underline{r}x\right)$ and find optimality conditions for the level of investment, x^*, that maximises the entrepreneur's expected utility $Eu(x)$. Demonstrate that x^* is increasing in π.

Question 3 (40%)

Suppose that, in order for the project described in Question 2 above to be realised, the entrepreneur requires support from an outside investor (the investor finances the remainder of the project, over and above what the entrepreneur puts in, x). If the project does not get support, the entrepreneur receives expected utility corresponding to an investment of zero, i.e. $x = 0$, $Eu(0) = v(0)$. The investor will support the project if and only if the likelihood of success is sufficiently high, i.e. $\tilde{\pi} > \pi^V$, where $\tilde{\pi}$ is the likelihood with which the investor believes the project will be a success and π^V is the threshold level for success.

Suppose that the project may be of one of two types, with high (H) and low (L) likelihood of success, respectively, i.e. $\pi_L < \pi_H$. We assume $\pi_L < \pi^V < \pi_H$.

A

Suppose both the entrepreneur and the investor know the type of the project. Demonstrate that, at equilibrium, the investor supports the project if and only if $\pi = \pi_H$ and that $x_L = 0$ and $x_H = x_H^*$, where, for $i = L, H$, x_i is the equilibrium investment choice of the entrepreneur if the project is of type i, and x_i^* is the investment that maximises expected utility derived in Question 2.B above when $\pi = \pi_i$.

B

Suppose next that only the entrepreneur knows the type of the project. The investor holds the prior belief that the project has a high likelihood of success (i.e. $\pi = \pi_H$) with probability α and a low likelihood of success (i.e. $\pi = \pi_L$) with probability $1 - \alpha$. Before the investor decides whether to support the project, the entrepreneur invests, i.e. chooses x. We denote by β the investor's posterior belief, having observed x, that the project has a high likelihood of success, while $1 - \beta$ denotes the investor's posterior belief that the project has low likelihood of success.

Let x_L^0 be the level of investment at which the entrepreneur is indifferent between investing and not investing when $\pi = \pi_L$, i.e. $Eu_L(x_L^0) = \pi_L v(\overline{r}x_L^0) + (1 - \pi_L)v(\underline{r}x_L^0) = Eu_L(0) = v(0)$.

We first consider the case that $x_L^0 > x_H^*$.

Explain that there exists a separating Perfect Bayesian equilibrium of the game in which the entrepreneur chooses $x_L = 0$ and $x_H = x_L^0$, and the investor believes $\beta = 0$ if $x < x_L^0$ and $\beta = 1$ if $x \geq x_L^0$ and invests only if $\beta = 1$.

C

Consider next the case that $x_L^0 < x_H^*$. Construct a pooling Perfect Bayesian equilibrium at which $x_L = x_H = x_H^*$. Explain why the existence of such an equilibrium requires that the investor expects that the project is a priori worth supporting, i.e. $\alpha\pi_H + (1 - \alpha)\pi_L > \pi^V$.

Question 4 (20%)

Explain how (costly) signalling may overcome problems of asymmetric information in markets for venture capital, like in the example in Question 3 above. Referring to the situation described in Question 3, under what conditions are such signalling likely to occur? Mention examples of other markets where similar signalling behaviour may occur.