Candidate instructions

ECON4310

This is some important information about the exam in ECON4310. Please read this carefully before you start answering the exam.

Date of exam: Monday, 16 December 2019

Time for exam: 2.30 p.m. - 5.30 p.m. (3 hours)

The problem set: The problem set consists of 4 questions with several sub-questions. They will count as indicated.

Sketches: You may use sketches on all questions. You are to use the sketching sheets handed to you. You can use more than one sketching sheet per question. See instructions for filling out sketching sheets below. It is very important that you make sure to allocate time to fill in the headings (the code for each problem, candidate number, course code, date etc.) on the sheets that you will use to add to your answer. You will find the code for each problem under the problem text. You will NOT be given extra time to fill out the "general information" on the sketching.

Access: You will not have access to your exam right after submission. The reason is that the sketches with equations and graphs must be scanned in to your exam. You will get access to your exam within 2-3 days.

Resources allowed: No written or printed resources - or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences).

Grading: The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Grades are given: January 6, 2020

1 Consider an economy in which each household has a utility function given by

$$\log(c_{1t}) + \beta \log(c_{2t+1}).$$

The household works for a wage w_t when young and has no labor income when old, but can save at a gross real interest rate $1 + r_{t+1}$. In each period a new generation is born whose size is $1 + n$ times as large as the previous generation. Thus the size of a generation born at time t is $N_t = (1 + n)^t$. For future reference define the per-capita (of the young generation) capital stock as $k_t = \frac{K_t}{N_t}$.

(a) **(a)**

10 points

Derive the optimal consumption and savings choice c_{1t}, s_t of the household.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(b) **(b)**

20 points

Suppose the wage is given by the marginal product of labor,

$$w_t = (1 - \alpha)(k_t)^{\alpha}$$

Use the market clearing condition in the capital market,

$$N_t s_t = K_{t+1}$$

to derive how the per capita capital stock k_{t+1} tomorrow depends on the per capita capital stock k_t today.

Fill in your answer here and/or on sketching paper

Maximum marks: 20

(c) **(c)**

10 points

Find the steady state capital stock(s) in the economy. How many steady states are there?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(d) # (d)

20 points

Now suppose the government levies a proportional tax $\tau > 0$ on labor income (wages), so that now income in the first period of a household's life is given by $(1-\tau)w_t$. The government uses the tax receipts to buy tanks that neither yield utility nor affect the production function. How is the steady state capital stock in the economy affected by this tax? Explain.

Fill in your answer here and/or on sketching paper

Maximum marks: 20

(e) # (e)

10 points

Consider the same situation as in question (d), but now suppose that the government returns the tax receipts to households as lump-sum transfers T_t. Thus income of the household is now

$$(1-\tau)w_t + T_t$$

Transfers in turn are given by tax receipts

$$T_t = \tau w_t.$$

How does your answer to question (d) change? Explain.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

2 Frodo lives for three periods. In the first period he acts in a movie and earns \$100, 000. In the second period he acts in a movie and earns \$150, 000 and in the last period of his life he acts in a movie and earns \$225, 000. His utility function is given by

$$u(c_1) + \tfrac{2}{3}u(c_2) + \tfrac{4}{9}u(c_3)$$

We assume, as always, $u'(c) > 0$ and $u''(c) < 0$. The market interest rate is $r = 50\%$ per period. He does not have any bequest from his parents $(A = 0)$.

(a) # (a)

10 points

Is Frodo's consumption profile increasing, decreasing or constant with age? Explain your answer.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(b) # (b)

10 points

Solve for the optimal consumption levels c_1, c_2, c_3 explicitly.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(c) # (c)

10 points

Solve for Frodo's savings in the first period of his life sav_1 and Frodo's financial asset position s_1 at the end of period 1. Recall that $s_0 = A = 0$ and that $r = 50\%$.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(d) # (d)

20 points

Now suppose Frodo cannot borrow. Determine the optimal consumption levels c_1, c_2, c_3, his optimal savings choices sav_1, sav_2, sav_3 and his asset positions s_1, s_2, s_3 over Frodo's life cycle.

Fill in your answer here and/or on sketching paper

Maximum marks: 20

3 Consider a household that chooses consumption *c* and labor supply *l* to solve the following maximization problem

$$\max_{c,l\geq 0}\left\{\log(c)-\frac{(l)^2}{2}\right\}$$

$$s.t.$$

$$c=(1-\tau)l+Tr$$

where τ is the labor income tax rate, Tr is a transfer by the government. Note that I set the wage per hour work to one to simplify your life.

(a) **(a)**

10 points

Derive the intratemporal optimality condition, relating labor supply and consumption.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(b) **(b)**

10 points

Suppose $Tr=0$. Solve for optimal labor supply.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(c) **(c)**

20 points

Use your answer in (a) to compare optimal labor supply in two scenarios

1. The tax revenues are thrown into the ocean and do not benefit the household at all, that is $Tr=0.$
2. The tax revenues are rebated back to the household, which yields $Tr>0$.

Under which scenario is labor supply higher, and why? You have to justify your answer even if you do not provide formal calculations.

Fill in your answer here and/or on sketching paper

Maximum marks: 20

4 Consider a household that potentially lives for two periods. Let p denote the probability that she survives to the second period. Her utility function is given by

$$\log(c_1) + p\log(c_2)$$

where c_1 is first period consumption and c_2 is second period consumption if the house-hold is alive in the second period. The household has income $y_1 = 10,000$ in the first period of life, but no labor income in the second period of life. Thus the budget constraints read as

$$c_1 + s = (1-\tau)10000$$
$$c_2 = (1+r)s + b$$

where τ is the social security tax rate and b are social security benefits. The population grows at rate 100% so that there are twice as many young people as there are old people. There is no income growth over time (that is $g = 0$).

(a)

(a)

10 points

Write down the budget constraint of the government, relating social security benefits b to taxes τ and the survival probability p.

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(b)

(b)

10 points

Suppose the survival probability is $p = 0.8$. What does the social security tax rate τ have to be so that the social security replacement rate is 40%?

Fill in your answer here and/or on sketching paper

Maximum marks: 10

(c)

(c)

10 points

Suppose that still $p = 0.8$ and that the interest rate is $r = 2 = 200\%$. What is the social security tax rate τ and associated benefit b the government should choose to maximize utility of the typical household in this economy. You need to explain your answer.

Fill in your answer here and/or on sketching paper

Maximum marks: 10