

The exam consists of three parts, A, B, and C, with equal weight (1/3). Remember to allocate your time accordingly.

## **Part A (1/3 of the exam): Essay**

Write a short essay addressing the following question in 500–750 words. In addressing the question, relate to the course literature.

*What are Friedman (1953)'s and Lucas (1976)'s criteria for a "good" model? In your opinion, which (or all, or neither) of these criteria do medium-sized DSGE models such as Norges Bank's model NEMO, as described in Gerdrup and Nicolaisen (2011), strive to satisfy? And which of these criteria do the SAM framework ("System for Averaging Models") strive to satisfy?*

### **References**

Friedman, M. (1953). The methodology of positive economics.

Gerdrup, K. R. and Nicolaisen, J. (2011). On the purpose of models - The Norges Bank experience. *Norges Bank Staff Memo*, (6).

Lucas, R. E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 1:19–46

## Part B (1/3 of the exam): Consumption Models

Consider the following household model

$$\max_{c_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

subject to

$$c_t + a_{t+1} = (1+r)a_t + y \text{ for all } t \quad (2)$$

where  $c_t$  is consumption in period  $t$ ,  $a_t$  is savings,  $y$  is income,  $r$  is the real interest rate, and  $\beta$  is the discount factor. Assume that  $\lim_{T \rightarrow \infty} \frac{a_{T+1}}{(1+r)^T} = 0$ , and  $u'(c) > 0$  and  $u''(c) < 0$  for any  $c \geq 0$ . Note that  $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r}$ .

- (a) Show how you can rewrite the budget constraints (2) into the present value budget constraint

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y}{(1+r)^t}. \quad (3)$$

- (b) Interpret the present value budget constraint (3).  
(c) Show that the solution to the problem has to satisfy

$$u'(c_t) = \beta(1+r)u'(c_{t+1}) \text{ for all } t. \quad (4)$$

- (d) Interpret equation (4).  
(e) Assume that  $\beta(1+r) = 1$ . Show that the consumption function is

$$c_t = ra_t + y. \quad (5)$$

- (f) Consider the case in problem (e) with  $\beta(1+r) = 1$ . What happens with consumption if  $a_t$  suddenly increases? What happens with consumption if  $y$  suddenly increases? Compute your answer and explain the intuition.

## Part C (1/3 of the exam): The Solow Model

In this exercise, we consider the Solow model. There is population growth ( $L_t = (1+n)L_{t-1}$ ) but no technological growth. Output  $Y_t$  is given by  $Y_t = F(K_t, L_t)$ , where  $F$  is an aggregate production function satisfying the usual (“neoclassical”) properties, where  $K_t$  is the capital stock and  $L_t$  is the labor force at time  $t$ . Capital depreciates at rate  $\delta \in (0, 1]$ , so we have  $K_{t+1} = (1 - \delta)K_t + I_t$ , where  $I_t$  is investment. Investment is a constant share of output,  $I_t = sY_t$  with  $s \in (0, 1)$ . Finally, the economy is closed (no trade with other economies), so  $Y_t = C_t + I_t$ , where  $C_t$  is consumption.

- (a) Define  $k_t = K_t/L_t$  to be the capital-labor ratio and define  $f(k) = F(k, 1)$ . Show that the law of motion for the capital-labor ratio is given by

$$k_{t+1} = \frac{1}{1+n} (sf(k_t) + (1-\delta)k_t).$$

- (b) In the long run, the capital-labor ratio will converge to a steady state value  $k^*$ . What is the long-run capital-to-output ratio? I.e., derive an expression for

$$\lim_{t \rightarrow \infty} \frac{K_t}{Y_t}$$

in terms of parameters. *Hint: what is the relationship between  $K_t/Y_t$  and  $k_t/y_t$ , where  $y_t = Y_t/L_t$  is output per capita?*

- (c) What is the long-run growth rate of  $Y_t$ ? What is the long-run growth rate of  $y_t$ ? (No need to show any work for this part.)
- (d) Now, assume  $F(K, L) = K^\alpha L^{1-\alpha}$  with  $\alpha \in (0, 1)$  and  $\delta = 1$ . Using the law of motion for the capital-labor ratio, show that

$$\frac{y_{t+1} - y_t}{y_t} = \left( \frac{s}{1+n} \right)^\alpha y_t^{\alpha-1} - 1$$

where  $y_t = Y_t/L_t$  is output per capita. What prediction does the above equation make about the relationship between GDP per capita growth and GDP per capita? *Hint: Use the law of motion for  $k_t$  with  $k_t$  re-expressed in terms of  $y_t$ .*