The exam consists of three parts, A, B, and C, with equal weight (1/3). Remember to allocate your time accordingly.

Part A (1/3 of the exam): Essay

Write a short essay addressing the following question in no more than 500 words. In addressing the question, relate to the course literature.

Solow (1997) wrote the following on the proper modelling strategy for short-run macroeconomic analysis:

> *My choices would be to model the main components of aggregate demand more or less opportunistically. By "opportunistically" I mean that whatever works (empirically). By "more or less" I mean I would want consumption functions, investment functions, import functions, and the like to look as if they could plausibly arise from aggregation of economic behavior of some reasonable kind at the micro level. That has always been the custom in macroeconomics, and I would not want to abandon it.*

Describe in what way a representative-agent model of consumption behavior does not "work" according to Solow's criterion. Discuss whether recent macroeconomic models ("HANK") satisfy Solow's "more or less opportunistic" approach, and in what way empirical work influences recent quantitative macroeconomic theory.

References

Solow, R. M. (1997). Is There a Core of Usable Macroeconomics We Should All Believe In? *American Economic Review*, 87(2):230–232.

Part B (1/3 of the exam): Investment Theory

Consider a project with a infinitely-lived cash flow d_{t+k} that is known in advance.

(a) Explain why the value V_0 of the project at time 0 with a constant interest rate is

$$V_0 = \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t d_t. \tag{1}$$

(b) Suppose the cash flow is constant across time, $d_{t+k} = d$ for all $k > 0$. Show that the value V of the project is also constant and equal to $V = \frac{d}{r}$.

Consider a firm that solves the following problem

$$\max_{\{d_t, K_{t+1}, I_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \left(\frac{1}{1+r} \right)^t d_t$$

subject to

$$\begin{aligned} d_t &= K_t^{\alpha} - I_t, \\ K_{t+1} &= (1-\delta)K_t + I_t, \end{aligned}$$

where d is earnings, K is capital, I is investment, r is the interest rate, and δ is the depreciation rate.

(c) Show that the optimal level of capital is constant and equal to

$$K_t = K^* = \left(\frac{\alpha}{r+\delta} \right)^{\frac{1}{1-\alpha}} \quad \text{for all } t > 0 \tag{2}$$

(d) Explain the intuition for how an increase in the depreciation rate δ affects the level of capital.

(e) Show that the value of the firm is

$$V_0 = \frac{\left(\frac{\alpha}{r+\delta} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left(\frac{\alpha}{r+\delta} \right)^{\frac{1}{1-\alpha}}}{r}. \tag{3}$$

(f) Explain intuitively through which channels a decrease in the interest rate r affects the value of the firm.

Part C (1/3 of the exam): Piketty

In his bestseller *Capital in the 21st century*, Thomas Piketty introduced what he termed the *second fundamental law of capitalism*:

$$\frac{K}{Y} = \frac{s}{g} \tag{4}$$

That is, the long-run capital-to-income ratio equals the savings rate divided by the growth rate of the economy. In this question, we will derive this law, explore its consequences, and compare it with what we covered in class.

Assume that capital accumulation is given by the following law of motion:

$$K_{t+1} = sY_t + K_t \tag{5}$$

where K_t is the capital stock at time t, Y_t is the income at time t, and s is the savings rate. That is, the capital stock tomorrow equals the capital stock today, K_t, plus investment today, sY_t.

(a) Assume further that the economy is on a balanced growth path such that $Y_{t+1} = (1+g)Y_t$ and $K_{t+1} = (1+g)K_t$. By manipulating Equation (5), show that the capital-to-income ratio K_t/Y_t satisfies the second fundamental law of capitalism, Equation (4).

(b) Piketty uses the second fundamental law of capitalism to forecast the long-run capital-to-income ratio if growth falls from $g = 0.02$ (2% growth rate) to $g = 0.01$ (1% growth rate), assuming that the savings rate remains constant. Describe, in at most three sentences, the consequences for the capital-to-income ratio of this change in growth rate.

(c) Equation (5) looks slightly different from what we covered in class. The capital accumulation equation in the Solow model was given by

$$K_{t+1} = sY_t + (1-\delta)K_t. \tag{6}$$

Explain the difference between Equation (5) and Equation (6). In particular, what does the parameter δ represent in the Solow model?

(d) Assume again that the economy is on a balanced growth path such that $Y_{t+1} = (1+g)Y_t$ and $K_{t+1} = (1+g)K_t$. By manipulating Equation (6), show that the capital-to-income ratio in the Solow model K_t/Y_t satisfies

$$\frac{K_t}{Y_t} = \frac{s}{g+\delta}.$$

(e) Assume that $\delta = 0.1$. Describe the effect of a fall in growth from $g = 0.02$ to $g = 0.01$ on the capital-to-income ratio in the Solow model, assuming that the savings rate remains constant. In particular, contrast with the forecast of Piketty's model.

(f) In the Solow model, define net income as $\hat{Y}_t = Y_t - \delta K_t$ and the net savings rate at time t as

$$\hat{s}_t = \frac{sY_t - \delta K_t}{\hat{Y}_t}.$$

Explain in words what net income and net savings rate are.

(g) Show that the Solow model can be rewritten on the net form

$$K_{t+1} = \hat{s}_t \hat{Y}_t + K_t.$$

The distinction between Piketty's model and the Solow model is thus a difference between whether we formulate our model in terms of net or gross income. However, they imply different theories of savings behavior. One predicts that the net savings rate is constant when growth falls, while the other predicts that the gross savings rate is constant when growth falls.