Final Exam
ECON 4310, Fall 2018

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	Max
Exercise A		**40**
Exercise B		**60**
Exercise C		**60**
Σ		**160**

Grade: ___________

Exercise A:
Short Questions (40 Points)

Answer each of the following short questions on a separate answer sheet. You will only get points for correct answer with an explanation.

Exercise A.1: (20 Points) Ricardian Equivalence

You are advisor to the Swedish king in 1618, right at the onset of the 30-year war in Europe between protestant forces, led by Sweden, and catholic forces, led by the German emperor. The cost, per capita, of the war for the next thirty years, is $10,000$ kroner. The king has come up with three policies to finance the war:

a Finance the war with immediate taxes of $10,000$ kroner.

b Issue government debt, and repay that debt, including interest, in the 30 year period after the war (1648-1678).

c Issue government debt and simply pay the interest on that government debt forever, without ever redeeming the debt itself.

Assume that the interest rate for a 30 year period is $r = 100\%$ (so that $1 + r = 2$).

Now consider Snorre Viking, a Swedish fisherman that lives from 1618 to 1678, that is, for 2 periods lasting 30 years each. By selling his fish he earns $15,000$ kroner in the first period of his life and $30,000$ kroner in the second period of his life (he gets better catching fish with experience). Snorre has utility function

$$\log(c_1) + \log(c_2)$$

A1.1 (6 points) Assume that Snorre can borrow and lend freely. How does Snorre rank policies a., b. and c., that is, which one does he like best and which one is worst for him? Explain.

Your Answer:

Snorre likes c. best (since most of the tax burden is transferred to future generations) and is indifferent between a. and b. since both policies imply the same present discounted value of taxes for Snorre.

A1.2 (7 points) Now suppose that Snorre cannot borrow any longer. How does Snorre rank policies a., b. and c., that is, which one does he like best and which one is worst for him? Explain.

Your Answer:

Answer: Policy c. is still best, but now (because of the binding borrowing constraint), he prefers b. to a.

A1.3 (7 points) Finally, suppose that Snorre eats so much healthy fish that he lives forever and that he can borrow. All other things remain the same. How does Snorre rank policies a., b. and c., that is, which one does he like best and which one is worst for him? Explain.

Your Answer:

Now he is indifferent among all three policies since all three have the same present discounted value of taxes.

Exercise A.2: (10 Points) Permanent Technology shocks in Real business cycle model and Consumption Response

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Consider a simple two-period model of labor supply, as we saw in lectures, where we assume that utility is separable in consumption and labor supply:

$$\begin{aligned}
\max_{\{c_0,c_1,h_0,h_1,a_1\}} \; & \log c_0 - \phi\frac{h_0^{1+\theta}}{1+\theta} + \beta[\log c_1 - \phi\frac{h_1^{1+\theta}}{1+\theta}] \\
s.t. \; & \\
c_0 + a_1 &= w_0 h_0 + (1+r_0)a_0 \\
c_1 &= w_1 h_1 + (1+r_1)a_1
\end{aligned}$$

for given $a_0 = 0$. Assume r_0, r_1 are exogenously given. We know the household has the following intertemporal labor supply condition:

$$\beta\frac{\phi h_1^{\theta}}{\phi h_0^{\theta}} = \frac{w_1}{(1+r_1)w_0},$$

and the solution for h_0 is given by:

$$\phi h_0^{1+\theta}\left[1+\left(\frac{w_1}{(1+r_1)w_0}\right)^{1+\frac{1}{\theta}}\beta^{-\frac{1}{\theta}}\right] = (1+\beta).$$

Suppose there is a permanent change to wages at the beginning of time 0: both wages in the first and second period increase by 10%. Then this household will take advantage of this opportunity consume more in c_0 by 10%. True or false?

Your Answer:

True ☐ False: ☒

By inspecting the solution for h_0 we know h_0 will not change; Then by using the intertemporal labor supply condition, we know h_1 and h_0 is proportional to each other. Therefore, labor supply h_1 does not change.

Going to consumption and saving, by looking at the Euler equation and the life-time budget constraint:

$$\frac{c_1}{c_0} = \beta(1+r_1), c_0 + \frac{c_1}{1+r_1} = w_0 h_0 + \frac{w_1 h_1}{1+r_1},$$

we know c_0 and c_1 will both increase by 10%.

Exercise A.3: (10 Points) Consumption and Saving with Worker Heterogeneity

Consider households' optimal intertemporal consumption choice in a two-period model. Suppose there are two types of workers in the economy, type A with constant wages w_A over time and type B with constant wages w_B, with $w_B = (1 + 10\%)w_A$. Both of them begin with 0 initial assets. Households have preferences $U = \sum_{t=0}^{1} \beta^t u(c_t)$ where $\beta \in (0,1)$ is the discount factor and the momentary utility function is

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}, \quad \theta > 1.$$

Also, assume the risk-free interest rate r is constant. Assume $\beta(1 + r) < 1$.

Denote the optimal consumption for type-A household as (c_0^A, c_1^A), and (c_0^B, c_1^B) for type-B household. Then we know type-B workers will have relatively lower consumption in the first period, i.e., $c_0^B < (1 + 10\%)c_0^A$.

True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

For each type of households, we should have the Euler equation and the life-time budget constraint satisfied:

$$\frac{c_1}{c_0} = (\beta(1+r))^{1/\theta}, c_0 + \frac{c_1}{1+r} = w + \frac{w}{1+r},$$

Since type-B workers have higher life-time wealth by 10%, we can see the consumption will also be higher than that for type-A workers; Moreover, they have the same consumption slope over time, so it must be that: $c_0^B = (1 + 10\%)c_0^A$

Exercise B:
Long Question (60 Points)
A Four Period Model

For the entire question, the interest rate is $r = 0$. First consider a household that lives for four periods. It has utility function

$$\log(c_1) + \log(c_2) + \log(c_3) + \log(c_4)$$

and income in the four periods of $y_1 = 10,000$, $y_2 = 10,000$, $y_3 = 50,000$ and $y_4 = 10,000$.

(a) (5 Points) Compute the optimal consumption choices (c_1, c_2, c_3, c_4).

Solution:

$$c_1 = c_2 = c_3 = c_4 = 20,000$$

(b) (10 Points) Suppose the household cannot borrow. Now what are the optimal consumption choices?

Solution:

The unconstrained optimal choice is not any longer feasible. The household want to equate consumption across periods as much as possible (since $(1+r)\beta = 1$). Thus the household consumes as much as possible in periods 1 and 2, $c_1 = c_2 = 10,000$. From period 3 onwards the household has income with present discounted value of $60,000$. It is optimal to split it equally, thus $c_3 = c_4 = 30,000$.

Now consider two members of the same dynasty that both live for two periods. Children have utility function

$$\log(c_3) + \log(c_4)$$

and parents have the utility function

$$\log(c_1) + \log(c_2) + V(b)$$

where b are the bequests left to the children and $V(b)$ is the maximal utility children can obtain when given bequests b. Income of parents is $(y_1, y_2) = (10,000, 10,000)$ and that of children is $(y_3, y_4) = (50,000, 10,000)$.

(c) (10 Points) Solve the maximization problem of the children to obtain $V(b)$, that is, solve

$$\begin{aligned} V(b) &= \max_{c_3,c_4} \log(c_3) + \log(c_4) \\ &\quad s.t. \\ c_3 + c_4 &= 60,000 + b \end{aligned}$$

Solution:

Answer: since $\beta(1+r) = 1$, we have

$$c_3 = c_4 = \frac{60,000 + b}{2}$$

and thus

$$V(b) = 2\log\left(\frac{60,000 + b}{2}\right)$$

(d) (15 Points) Use your answer from the previous question to solve the parents' maximization problem. *Allow bequests to be negative.*

Solution:

the maximization problem is

$$\begin{aligned} &\max_{c_1,c_2,b} \log(c_1) + \log(c_2) + 2\log\left(\frac{60,000 + b}{2}\right) \\ &\text{s.t.} \\ c_1 + c_2 + b &= 20,000 \end{aligned}$$

The Lagrangian is

$$L = \log(c_1) + \log(c_2) + 2\log\left(\frac{60,000 + b}{2}\right) + \lambda\left(20,000 - c_1 - c_2 - b\right)$$

First order conditions are

$$\begin{aligned} \frac{1}{c_1} &= \lambda \\ \frac{1}{c_2} &= \lambda \\ \frac{2}{60,000 + b} &= \lambda \end{aligned}$$

Thus

$$c_1 = c_2 = \frac{60,000 + b}{2}$$

Using this in the budget constraint yields

$$\begin{aligned} 60,000 + 2b &= 20,000 \\ b &= -20,000 \end{aligned}$$

and thus

$$c_1 = c_2 = 20,000$$

(e) (10 Points) Repeat question 5., but now assume that bequests cannot be negative (that is, assume a constraint of the form $b \geq 0$). You don't have to do any calculations, but you have to explain your answer.

Solution:

From the previous questions we know the households would like to smooth consumption, but also that the old generation would like to leave negative bequests. If that is not permitted, then $b = 0$ is optimal and the optimal consumption choices are

$$\begin{aligned} c_1 &= c_2 = 10,000 \\ c_3 &= c_4 = 30,000 \end{aligned}$$

exactly as in question 2.

(f) (10 Points) Bequests still cannot be negative. Now suppose the government increases taxes in period 2 by 5,000 and gives back 5,000 in subsidies in period 3. What is the optimal consumption allocation now. Again you don't have to do any calculations, but you have to explain your answer.

Solution:

Now generation 1 has lifetime income of 15,000 and generation 2 of 65,000. Generation 1 still cannot leave negative bequests. Thus the optimal consumption choices are

$$\begin{aligned} c_1 &= c_2 = 7,500 \\ c_3 &= c_4 = 32,500 \end{aligned}$$

Exercise C:
Long Question (60 Points)

A real business cycle model

Consider a representative household of a closed economy. The household has a planning horizon of two periods and is endowed with the following preferences over consumption, c,

$$U = u(c_1) + \beta \mathrm{E} u(c_2(s_2)),$$

with the following marginal utility

$$u'(c) = c^{-\gamma},\ \gamma \geq 1.$$

The variable s_2 denotes the state of the economy in the second period which follows the stochastic process

$$s_2 = \begin{cases} s_G, \text{ with prob. } p \\ s_B, \text{ with prob. } 1-p, \end{cases}$$

and the household conditions the consumption, $c_2(s_2)$, in the second period on the state, s_2. Assume the household's labor supply is exogenous and always equal to 1.

Labor market assumptions:

Assume that in each period and in each state of the economy, s_t, there is a linear (in labor n_t) production technology of the form

$$y_t(s_t) = A_t(s_t) n_t(s_t),$$

and the labor market is assumed to be competitive. Assume the labor productivity in the first period is given by $A_1 = A$, and the labor productivity is higher in the good state of the second period,

$$A_2(s_G) = A + A(1-p)\epsilon > A_2(s_B) = A - Ap\epsilon, \quad \epsilon > 0, A > 0, 0 < p < 1,$$

than in the bad state of the second period. The wages are denoted as w_1, $w_2(s_G)$, and $w_2(s_B)$.

Asset market assumptions:

Assume the household does have access to a risk-free asset, a_2, and the associated interest rate is denoted as r_2.

(a) (5 Points) Find the equilibrium wages, w_1, $w_2(s_G)$, and $w_2(s_B)$, and show that the expected wages in the second period is the same as wage in in the first period.

Solution:

Since labor markets are competitive, we should have wages equal to productivity as follows:

$$\begin{aligned} w_1 &= A, \\ w_2(s_G) &= A_2(s_G), \\ w_2(s_B) &= A_2(s_B). \end{aligned}$$

***XX Allocation of points**:*
3 points for get the wages correct ; 2 points for the proof is correct

(b) (5 Points) Write down the state-by-state budget constraints for the household.

Solution:

the state-by-state budget constraints for the household are:

$$\begin{aligned} c_1 + a_2 &= w_1, \\ c_2(s_2) &= w_2(s_2) + (1 + r_2)a_2, \ \forall s_2 \in S \equiv \{s_G, s_B\}. \end{aligned}$$

***XX Allocation of points**:*
1 point for c_1; 2 points for each of the other quantities

(c) (10 Points) Let $(\lambda_1, \lambda_2(s_G), \lambda_2(s_B))$ denote the Lagrange multipliers of the state-by-state budget constraints. State the representative agent's Lagrangian. (Note that the expected utility for the second period is the summation of utility across good and bad states, weighted by probability, i.e., $\mathrm{E}u(c_2(s_2)) = pu(c_2(s_G)) + (1 - p)u(c_2(s_B))$.)

Solution:

The state-by-state budget constraints are:

$$\begin{aligned} c_1 + a_2 &= w_1 \times 1, \\ c_2(s_2) &= w(s_2) \times 1 + (1 + r_2)a_2, \ \forall s_2 \in S \equiv \{s_G, s_B\}. \end{aligned}$$

The Lagrangian can be written in the state-ordered form as

$$\begin{aligned} \mathcal{L} = u(c_1) &+ \lambda_1 \left[w_1 - a_2 - c_1\right] \\ &+ \beta p \left[u(c_2(s_G))\right] + \lambda_2(s_G) \left[w(s_G) + (1 + r_2)a_2 - c_2(s_G))\right] \\ &+ \beta(1 - p) \left[u(c_2(s_B))\right] + \lambda_2(s_B) \left[w(s_B) + (1 + r_2)a_2 - c_2(s_B))\right]. \end{aligned}$$

***XX Allocation of points**:*
10 points for the Lagrangian: if correct or make sense (could be in other formulations); deduct 3 points if one of the three blocks wrong.

(d) (10 Points) Derive the optimality conditions with respect to consumption, $(c_1, c_2(s_G), c_2(s_B))$ and savings, a_2 by using multipliers.

Solution:

The optimality conditions with respect to the choices are:

$$0 = \frac{\partial \mathcal{L}}{\partial c_1} = u'(c_1) - \lambda_1 \tag{1}$$

$$0 = \frac{\partial \mathcal{L}}{\partial c_2(s_G)} = \beta p u'(c_2(s_G)) - \lambda_2(s_G) \tag{2}$$

$$0 = \frac{\partial \mathcal{L}}{\partial c_2(s_B)} = \beta(1-p)u'(c_2(s_B)) - \lambda_2(s_B) \tag{3}$$

$$0 = \frac{\partial \mathcal{L}}{\partial a_2} = -\lambda_1 + [\lambda_2(s_G) + \lambda_2(s_B)](1+r_2). \tag{4}$$

***XX Allocation of points**:*
Deduct 2.5 point if the FOC for one of the choices wrong.

(e) (10 Points) Derive the stochastic consumption Euler equation (only involves with $c_1, c_2(s_2), \beta$ and r_2 and No multipliers).

Solution:

The stochastic consumption Euler equation is given by:

$$u'(c_1) = \beta \mathrm{E}\left[u'(c_2(s_2))\right](1+r_2). \tag{5}$$

***XX Allocation of points**:*
10 points if the final formula correct; otherwise, 0 points.

(f) (10 Points) For (f) and (g), assume that the asset a_2 is available in zero supply.

What is the household's optimal choice of a_2 in the equilibrium? What are the household's optimal choices of consumption? Can the household fully smooth consumption? i.e., are c_1, $c_2(s_G)$ and $c_2(s_B)$ equal?

Solution:

In equilibrium $a_2 = 0$ since we assume this is a representative household and zero net asset supply. The state-by-state budget constraints imply the following consumption levels

$$c_1 = w_1$$
$$c_2(s_2) = w(s_2),\ \forall s_2 \in S,$$

or,

$$c_1 = w_1 = A,$$
$$c_2(s_G) = w_2(s_G) = A_2(s_G) > A,$$
$$c_2(s_B) = w_2(s_B) = A_2(s_B) < A.$$

so consumptions are not fully smoothed.

XX Allocation of points*:*
4 points to get the savings correct. 2 points for each of the consumption quantities.

(g) (10 Points) Is the equilibrium interest rate r_2 higher or lower than $r_{RN} \equiv \frac{1}{\beta} - 1$? Why? (Hint: do it step by step: (1) use the budget constraint to link consumption and wages; (2) use the Euler equation and the result, $u'(w_1) \leq \mathrm{E}[u'(w(s_2))]$, which comes from the Jensen's inequality.)

Solution:

In the stochastic economy with $\epsilon > 0$, Jensen's inequality implies

$$(1 + r_2)\beta = \frac{u'(c_1)}{\mathrm{E}\left[u'(c(s_2))\right]} = \frac{u'(w_1)}{\mathrm{E}\left[u'(w(s_2))\right]} = \frac{u'(\mathrm{E}\left[w(s_2)\right])}{\mathrm{E}\left[u'(w(s_2))\right]} < 1,$$

such that the interest rate in this economy is smaller than the risk neutral interest rate, $r_{RN} \equiv \frac{1}{\beta} - 1$.

XX Allocation of points*:*

(1) 5 points: get the budget constraint and use the Euler equation correctly

(2) 5 points: use the inequality and compare the two interest rates correctly