The exam consists of three parts, A, B, and C, with equal weight (1/3). Remember to allocate your time accordingly.

Part A (1/3 of the exam): Essay

Write a short essay addressing the following question in 500–750 words. In addressing the question, relate to the course literature.

What are Friedman (1953)'s and Lucas (1976)'s criteria for a "good" model? In your opinion, which (or both, or neither) of these criteria do medium-sized DSGE models such as Norges Bank's model NEMO, as described in Gerdrup and Nicolaysen (2011), strive to satisfy? How about the SAM framework ("System for Averaging Models")?

References

Friedman, M. (1953). The methodology of positive economics.

Gerdrup, K. R. and Nicolaisen, J. (2011). On the purpose of models - The Norges Bank experience. *Norges Bank Staff Memo*, (6).

Lucas, R. E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 1:19–46

Part B (1/3 of the exam): Consumption-Saving Models

Consider the following two-period household model

$$\max_{c_1, b_1, c_2} \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma}$$
$$\text{subject to}$$
$$c_1 + b_1 = y$$
$$c_2 = (1+r)b_1$$

where c is consumption, y is income, b is savings, $\sigma > 0$ is a parameter, and r is the interest rate.

1. Show that the Euler equation is $c_1^{-\sigma} = (1+r)c_2^{-\sigma}$.

2. Explain why consumption growth depends on the interest rate r.

3. Show that consumption in period 1 is $c_1 = \frac{y}{1+(1+r)^{1/\sigma - 1}}$.

4. Explain intuitively why the effect of a change in the interest rate r on c_1 is ambiguous and depends on $(1/\sigma - 1)$. Why is the effect negative if $1/\sigma > 1$?

Consider a modified version of the model

$$\max_{\{c_1, b_1, c_2\}} \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\gamma}}{1-\gamma}$$
$$\text{subject to}$$
$$c_1 + b_1 = y$$
$$c_2 = b_1$$

where c is consumption, y is income, b is savings, and $\sigma > 0$ and $\gamma > 0$ are parameters. Note that there are two differences compared with the problem above: (i) consumption in period 2 has a different curvature parameter and (ii) there is no interest rate ($r = 0$).

5. Show that $c_2 > c_1$ if $\sigma > \gamma$ and explain why the growth of consumption (c_2 relative to c_1) now depends on σ relative to γ.

Part C (1/3 of the exam): The Solow Model

In this exercise, we consider the Solow model. There is no technological growth and no population growth. Output Y_t is given by $Y_t = F(K_t, L)$ where F is an aggregate production function satisfying the usual ("neoclassical") properties, where K_t is capital at time t and L is the (constant) labor force. Capital depreciates at rate $\delta > 0$, so we have $K_{t+1} = I_t + (1-\delta)K_t$. Investment is a constant share of output, $I_t = sY_t$. The economy is closed (no trade with other economies) so $Y_t = C_t + I_t$

1. Given these assumptions, show that the law of motion for capital is given by $K_{t+1} = sF(K_t, L) + (1-\delta)K_t$.

2. In the long run, the capital stock converges to its steady state value K_{ss}. Show that the steady-state capital-output ratio is $K_{ss}/Y_{ss} = s/\delta$.

3. Now, we consider the special case with $F(K_t, L) = K_t^\alpha L^{1-\alpha}$. For simplicity, assume $L = 1$, so $F(K_t, L) = K_t^\alpha$ with $\alpha > 0$. Show that steady-state capital is given by $K_{ss} = (s/\delta)^{1/(1-\alpha)}$ and steady-state output is given by $Y_{ss} = (s/\delta)^{\alpha/(1-\alpha)}$.

4. If the saving rate s increases, does steady-state output Y_{ss} necessarily increase? Provide a mathematical argument for your conclusion.

5. Steady-state consumption is given by $(1-s)Y_{ss}$. If the saving rate s increases, does steady-state consumption C_{ss} necessarily decrease? Provide an argument for your conclusion (which need not be mathematical).

Solution Proposal

Part A

Here are criteria for answering part A well.

1. The student should demonstrate an understanding of the main criteria of a "good" model in (positive) economics in Friedman (1953).
2. The student should demonstrate an understanding what the Lucas critique is and what Lucas' criteria of a "good" model is (Lucas, 1976).
3. The student should demonstrate an understanding of Gerdrup and Nicolaysen (2011).
   (a) The student should provide a brief explanation of what a medium-sized DSGE model is (e.g., NEMO) and explain how it satisfies/does not satisfy the criteria in Friedman (1953) and Lucas (1976).
   (b) The student should provide a brief explanation of what SAM is and explain how it satisfies/does not satisfy the criteria in Friedman (1953) and Lucas (1976).
4. The essay should be well-structured and well-written.

If all four criteria are satisfied, the student should get a full score.

Part B

1. Example solution by Lagrange:

$$\mathcal{L} = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\sigma}}{1-\sigma} - \lambda_1(c_1 + b_1 - y) - \lambda_2(c_2 - (1+r)b_1)$$

   FOCs:

$$\frac{\partial \mathcal{L}}{\partial c_1} = c_1^{-\sigma} - \lambda_1 = 0$$
$$\frac{\partial \mathcal{L}}{\partial c_2} = c_2^{-\sigma} - \lambda_2 = 0$$
$$\frac{\partial \mathcal{L}}{\partial b_1} = -\lambda_1 + (1+r)\lambda_2 = 0$$

   which together give

$$c_1^{-\sigma} = (1+r)c_2^{-\sigma}.$$

2. The interest rate is the price of goods in period 1 relative to period 2. If the interest rate is above 0 in this case, consumption is relatively cheaper in period 2 and the household will move resources to period 2 by saving in period 1 and consuming later. Consumption growth, c_1/c_2 is therefore increasing in the interest rate.

3. The euler equation implies that

$$c_2 = (1+r)^{1/\sigma} c_1.$$

   Combine the two budget constraints to get

$$c_1 + \frac{c_2}{1+r} = y.$$

   Combining the euler and the budget constraints, we get

$$c_1 + (1+r)^{1/\sigma - 1} c_1 = y$$

   which becomes

$$c_1 = \frac{y}{1 + (1+r)^{1/\sigma - 1}}.$$

4. The effect of an interest rate change on the c_1 is ambiguous. There are two effects, an income and a substitution effect. A higher interest rate raises the price of con-

sumption in period 1 relative to period 2, resulting in a reduction in consumption in period 1. The income effect, on the other hand, works in the opposite direction. Because the household has income only in period 1, it is originally planning to save. A higher interest rate makes its consumption plan cheaper, and the household is therefore richer, resulting in higher consumption in period 1 and 2.

$1/\sigma$ governs the strength of the substitution effect, while 1 is the strength of the income effect. If $1/\sigma > 1$, the substitution effect dominates and the household reduces consumption in period 1.

5. The euler equation in the modified solution implies that

$$c_1^{-\sigma} = c_2^{-\gamma}.$$

This Euler equation implies that both $c_1 > c_2$ and $c_2 > c_1$ can be true and the correct answer depends on the level of y. The problem specifically asks for showing that $c_2 > c_1$ if $\sigma > \gamma$, which is true if $y > 2$. The student can show this using any method. For example, one can point out that for a given constant consumption, say 10, the marginal utility from consumption in period 1 is lower than consumption in period 2, and the agent would want to move resources from c_1 to c_2, thus $c_2 > c_1$. Because the dependence on y is not mentioned in the problem, the problem should be graded generously.

Part C

1. The law of motion for capital in the Solow growth model is derived using the capital accumulation formula where capital next period is equal to investment in the current period plus the remaining capital after depreciation:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

and replace for I_t

$$K_{t+1} = sY_t + (1 - \delta)K_t$$

which is equal to

$$K_{t+1} = sF(K_t, L) + (1 - \delta)K_t.$$

2. In the steady state, we have that $K_{t+1} = K_t = K_{ss}$ and $Y_{ss} = F(K_{ss}, L)$, which implies

that the law of motion of capital becomes

$$\begin{aligned} K_{ss} &= sY_{ss} + (1-\delta)K_s s \\ \delta K_{ss} &= sY_{ss} \\ K_{ss}/Y_{ss} &= s/\delta \end{aligned}$$

3. Using the Cobb-Douglas production function with $L = 1$, we get

$$sK_{ss}^{\alpha} = \delta K_{ss}$$

which implies that

$$K_{ss} = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

and thus that

$$Y_{ss} = K_{ss}^{\alpha} = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

4. We know that

$$Y_{ss} = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

taking the derivative with respect to the saving rate, we get

$$\frac{\partial Y_{ss}}{\partial s} = \frac{\alpha}{1-\alpha}\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{1}{\delta} = \frac{\alpha}{1-\alpha}\left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{s}$$

which is always positive if $\alpha \leq 1$. Note that the bounds for α was not specified on the exam so that we also accept answers where it is allowed that $\alpha > 1$.

5. Steady state consumption is

$$C_{ss} = (1-s)Y_{ss}$$

and the derivative with respect to the s is

$$\frac{\partial C_{ss}}{\partial s} = \frac{\alpha}{1-\alpha}\frac{1-s}{s}Y_{ss} - Y_{ss}$$

The first term is the effect that a higher saving rate has on higher output and thus consumption, and the second term is the direct effect of consuming less due to a higher saving rate. Again, since $\alpha \leq 1$ was not specified, we also accept answers where it is allowed that $\alpha > 1$.