The exam consists of three parts, A, B, and C, with equal weight (1/3). Remember to allocate your time accordingly.

Part A (1/3 of the exam): Essay

Write a short essay addressing the following question in 500–750 words. In addressing the question, relate to the course literature.

What are Friedman (1953)'s and Lucas (1976)'s criteria for a "good" model? In your opinion, which (or all, or neither) of these criteria do medium-sized DSGE models such as Norges Bank's model NEMO, as described in Gerdrup and Nicolaisen (2011), strive to satisfy? And which of these criteria do the SAM framework ("System for Averaging Models") strive to satisfy?

References

Friedman, M. (1953). The methodology of positive economics.

Gerdrup, K. R. and Nicolaisen, J. (2011). On the purpose of models - The Norges Bank experience. *Norges Bank Staff Memo*, (6).

Lucas, R. E. (1976). Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 1:19–46

Part B (1/3 of the exam): Consumption Models

Consider the following household model

$$\max_{c_{t_{t=0}^{\infty}}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

subject to

$$c_t + a_{t+1} = (1+r)a_t + y \text{ for all } t \tag{2}$$

where c_t is consumption in period t, a_t is savings, y is income, r is the real interest rate, and β is the discount factor. Assume that $\lim_{T\to\infty} \frac{a_{T+1}}{(1+r)^T} = 0$, and $u'(c) > 0$ and $u''(c) < 0$ for any $c \geq 0$. Note that $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r}$.

(a) Show how you can rewrite the budget constraints (2) into the present value budget constraint

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y}{(1+r)^t}. \tag{3}$$

(b) Interpret the present value budget constraint (3).

(c) Show that the solution to the problem has to satisfy

$$u'(c_t) = \beta(1+r)u'(c_{t+1}) \text{ for all } t. \tag{4}$$

(d) Interpret equation (4).

(e) Assume that $\beta(1+r) = 1$. Show that the consumption function is

$$c_t = ra_t + y. \tag{5}$$

(f) Consider the case in problem (e) with $\beta(1+r) = 1$. What happens with consumption if a_t suddenly increases? What happens with consumption if y suddenly increases? Compute your answer and explain the intuition.

Part C (1/3 of the exam): The Solow Model

In this exercise, we consider the Solow model. There is population growth ($L_t = (1+n)L_{t-1}$) but no technological growth. Output Y_t is given by $Y_t = F(K_t, L_t)$, where F is an aggregate production function satisfying the usual ("neoclassical") properties, where K_t is the capital stock and L_t is the labor force at time t. Capital depreciates at rate $\delta \in (0, 1]$, so we have $K_{t+1} = (1-\delta)K_t + I_t$, where I_t is investment. Investment is a constant share of output, $I_t = sY_t$ with $s \in (0, 1)$. Finally, the economy is closed (no trade with other economies), so $Y_t = C_t + I_t$, where C_t is consumption.

(a) Define $k_t = K_t/L_t$ to be the capital-labor ratio and define $f(k) = F(k, 1)$. Show that the law of motion for the capital-labor ratio is given by

$$k_{t+1} = \frac{1}{1+n}\left(sf(k_t) + (1-\delta)k_t\right).$$

(b) In the long run, the capital-labor ratio will converge to a steady state value k^*. What is the long-run capital-to-output ratio? I.e., derive an expression for

$$\lim_{t\to\infty} \frac{K_t}{Y_t}$$

in terms of parameters. *Hint: what is the relationship between K_t/Y_t and k_t/y_t, where $y_t = Y_t/L_t$ is output per capita?*

(c) What is the long-run growth rate of Y_t? What is the long-run growth rate of y_t? (No need to show any work for this part.)

(d) Now, assume $F(K, L) = K^\alpha L^{1-\alpha}$ with $\alpha \in (0, 1)$ and $\delta = 1$. Using the law of motion for the capital-labor ratio, show that

$$\frac{y_{t+1} - y_t}{y_t} = \left(\frac{s}{1+n}\right)^\alpha y_t^{\alpha-1} - 1$$

where $y_t = Y_t/L_t$ is output per capita. What prediction does the above equation make about the relationship between GDP per capita growth and GDP per capita? *Hint: Use the law of motion for k_t with k_t re-expressed in terms of y_t.*

Solution Proposal

Part A

Here are criteria for answering part A well.

1. The student should demonstrate an understanding of the main criteria of a "good" model in (positive) economics in Friedman (1953):

   - Models should be "evaluated based on its predictive power for the class of phenomena which it is intended to explain."
   - Because there is an infinite amount of models that can explain a limited set of empirical phenomena, we would want to choose the simplest among the models that can explain it.
   - Models cannot be discarded based on the falsity of their assumptions.

2. The student should demonstrate an understanding what the Lucas critique is and what Lucas' criteria of a "good" model is (Lucas, 1976):

   - "[T]he features which lead to success in short-term forecasting are unrelated to quantitative policy evaulations." Simulations using major econometric models are therefore not suitable for policy evaluations because the economic relationships may be affected by the policy change.
   - Models used for policy evaluation should therefore generally allow for agents in their model that respond to the policy change.

3. The student should demonstrate an understanding of Gerdrup and Nicolaisen (2011), which should include:

   (a) The student should provide a brief explanation of what a medium-sized DSGE model is (e.g., NEMO) and explain how it satisfies/does not satisfy the criteria in Friedman (1953) and Lucas (1976).

   (b) The student should provide a brief explanation of what SAM is and explain how it satisfies/does not satisfy the criteria in Friedman (1953) and Lucas (1976).

4. The essay should be well-structured and well-written.

If all four criteria are satisfied, the student should get a full score.

Part B

(a) Rearrange the period-by-period budget constraints and replace iteratively. Use that we assume $\lim_{T\to\infty} \frac{a_{t+1}}{(1+r)^T} = 0$ to remove it from the equation.

(b) Net present value of lifetime consumption is equal to the net present value of lifetime income. Lifetime income consists of any initial wealth and the net present value of current and future labor income.

(c) This problem can be solved in many ways. For example, set up the Lagrangian with the present value budget constraint as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) - \lambda \left(\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} - (1+r)a_0 - \sum_{t=0}^{\infty} \frac{y}{(1+r)^t} \right)$$

with the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t u'(c_t) - \frac{\lambda}{(1+r)^t} = 0 \text{ for all } t$$

where if you pick t and $t+1$, you get $u'(c_t) = \beta(1+r)u'(c_{t+1})$.

(d) Marginal utility of spending today has to be equal to the discount marginal utility from saving that unit and spending it tomorrow. Trade-off is spend vs. save.

(e) If $\beta(1+r) = 1$, we have $u'(c_t) = u'(c_{t+1})$, implying that $c_t = c_{t+1} = c$. Inserting this into the present-value budget constraint, one gets

$$\sum_{t=0}^{\infty} \frac{c}{(1+r)^t} = (1+r)a_0 + \sum_{t=0}^{\infty} \frac{y}{(1+r)^t} \Rightarrow \frac{1+r}{r}c = (1+r)a_0 + \frac{1+r}{r}y.$$

which implies that $c = ra_0 + y$. Since $a_{t+1} = a_t$, we also have that $c_t = ra_t + y$.

(f) If a_t increases, consumption increases by r times that amount. If y increases (permanently), consumption increases with the full amount of this income increases. If income increases temporarily (for one period), it is like an increase in a_t. This is the permanent income hypothesis. The consumption function is consistent with how much one could increase consumption forever. For a transitory increase (wealth or income), the spending response is the real return to that 'shock.' For permanent increases, consumption increases one-for-one with the 'shock' because the household is able to increase consumption forever with that amount.

Part C

(a) From the setup, we have

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{6}$$

$$I_t = sY_t, \tag{7}$$

$$Y_t = F(K_t, L_t). \tag{8}$$

Plugging in Equation (8) into Equation (7) and subsequently into Equation (6) yields

$$K_{t+1} = sF(K_t, L_t) + (1-\delta)K_t.$$

Dividing both sides by $L_{t+1} = (1+n)L_t$ yields

$$\frac{K_{t+1}}{L_{t+1}} = \frac{s}{1+n}F\left(\frac{K_t}{L_t}, 1\right) + \frac{1-\delta}{1+n}\frac{K_t}{L_t}$$

where we used that F is constant returns to scale. Finally, recalling the definitions of k_t and $f(k)$, we have

$$k_{t+1} = \frac{1}{1+n}\left(sf(k_t) + (1-\delta)k_t\right).$$

(b) In the steady state, we have $k_{t+1} = k_t = k^*$. Then, from the law of motion for the capital-labor ratio, we have

$$k^* = \frac{1}{1+n}\left(sf(k^*) + (1-\delta)k^*\right)$$

or

$$\frac{k^*}{f(k^*)} = \frac{s}{\delta+n}$$

Now, finally, we have $K_t/Y_t = (K_t/L_t)/(Y_t/L_t) = k_t/y_t$, so

$$\lim_{t\to\infty}\frac{K_t}{Y_t} = \lim_{t\to\infty}\frac{k_t}{y_t} = \frac{k^*}{y^*} = \frac{s}{\delta+n}.$$

(c) The long-run growth rate of Y_t is n and the long-run growth rate of y_t is 0.

(d) Since $y_t = k_t^\alpha$, we have $k_t = y_t^{1/\alpha}$. Then, from the law of motion for the capital-labor ratio, we have

$$y_{t+1}^{1/\alpha} = \frac{s}{1+n} y_t$$

where we used that $\delta = 1$. Equivalently, $y_{t+1} = \left(\frac{s}{1+n}\right)^{\alpha} y_t^{\alpha}$. Dividing both sides by y_t yields

$$\frac{y_{t+1}}{y_t} = \left(\frac{s}{1+n}\right)^{\alpha} y_t^{\alpha-1}.$$

Finally, subtracting 1 from both sides yields

$$\frac{y_{t+1} - y_t}{y_t} = \left(\frac{s}{1+n}\right)^{\alpha} y_t^{\alpha-1} - 1.$$

Since $\alpha - 1 < 0$, $y_t^{\alpha-1}$ is a decreasing function of y_t. Therefore, the above equation implies that GDP per capita growth is a decreasing function of GDP per capita.